NOTES
SUBJECT: MACHINE DESIGN-1
SUBJECT CODE: EME-501
BRANCH: MECHANICAL ENGINEERING
SEM: 5th
SESSION: 2014-15

Evaluation scheme:

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UNIT- I
**Introduction to Engineering Design:**
Engineering design may be defined as the iterative decision making activity to create a plan or plans by which the available resources are converted, preferably optimally, into systems, processes or devices to perform the desired functions and to meet human needs. In fact engineering design has been defined in many ways but the simplest ways to define engineering design as

“An iterative decision making process to conceive and implement optimum systems to solve society’s problems and needs.”

Engineering design is practical in nature and must be concerned with physical realizability, or economic and financial feasibility.

**Mechanical Engineering Design:**
If the end product of the engineering design can be termed as mechanical then this may be termed as Mechanical Engineering Design. Mechanical Engineering Design may be defined as:

“Mechanical Engineering Design is defined as iterative decision making process to describe a machine or mechanical system to perform specific function with maximum economy and efficiency by using scientific principles, technical information, and imagination of the designer.”

A designer uses principles of basic engineering sciences, such as Physics, Mathematics, Statics, Dynamics, Thermal Sciences, Heat Transfer, Vibration etc.

For example:
- Newton’s Laws of Motion;
- D’ Alembert’s Principle;
- Boyle’s and Charles Laws of Gases;
- Carnot Cycle;
- Bernoulli’s Principle;

The designer has technical information of basic elements of machines, like fastening devices, chain, belt and gear drives, bearings, oil seals etc. The designer knows or having the information about the relative advantages and disadvantages of these basic elements and their suitability in different applications.

**MACHINE DESIGN:**
Machine Design or mechanical design is primarily concerned with the systems by which the energy is converted into useful mechanical forms and of mechanisms required to convert the output of the machine to the desired form. The design may lead to an entirely new machine or an improvement on an existing one.

Thus machine design is the production or creation of the right combination of correctly proportioned moving and stationary components so constructed and joined as to enable the liberation, transformation, and utilization of energy.

The basic procedure of machine design (Mechanical Engineering Design) consists of a step by step approach from given specifications of functional requirement of a product to the
complete description in the form of blue prints of the final product. The following steps are involved:

**First Step:**
In the very first step a complete list of specifications for the functional requirement of the product is to be prepared. The requirement may include, for example:
- Output capacity;
- Service life;
- Cost;
- Reliability; etc.

In consumer products, in addition appearance, noiseless operation, and simplicity in control are important requirements.

Depending upon the type of product, various requirements are given weightage and a priority list of specifications is prepared.

**Second Step:**
After a careful study of the requirements the designer prepares rough sketches of different possible mechanisms of machine and depending upon the cost competitiveness, availability of raw material, and manufacturing facilities, the possible mechanisms are compared with each other and the designer selects the best possible mechanism for the product. For example for designing the Blanking and Piercing Press the following mechanisms may be thinking of:

1. Mechanism involving the **CRANK AND CONNECTING ROD** converting the rotary motion of the electric motor into reciprocating motion of the punch.
2. Mechanism involving **NUT AND SCREW** which results in simple and cheap configuration but having poor efficiency.
3. Mechanism involving **HYDRAULIC CYLINDER, PISTON AND VALVES**, which is a costly configuration but highly efficient.

![Figure 1.1](image_url)
In the third step of the design procedure a block diagram is to be prepared which showing the general layout of the selected configuration.
In this step designer specifies the joining methods, such as riveting, bolting, and welding to connect the individual components. Rough sketches of shapes of individual parts are prepared.
For example: the layout of an E.O.T (Electrically Operated Overhead Crane) will consist following components.

- Electric motor for supply of the power;
- Flexible coupling to connect the motor shaft to the clutch shaft
- Clutch to connect or disconnect the electric motor at the will of the operator.
- Gear box to reduce the speed from 1440 rpm to 15 rpm
- Rotary drum to convert the rotary motion of the shaft to the translator motion of the wire rope
- Wire rope and pulley with the crane hook to attach the load
- Brake to stop the motion.

Fourth Step: After selecting the required or deciding the configuration of mechanism / machine in third step above. The design of individual components of the selected configuration is to be done in this step. It consists of the following stages:

- Determine the forces acting on each component;
- Selecting the proper material for the component depending upon the functional requirement, such as strength, wear, rigidity, hardness and bearing properties etc.
- Determine the likely mode of failure & select the criterion of failure like, yield strength, ultimate strength, deflection etc.
- Determine the geometric dimensions of the components using suitable factor of safety and modify the dimensions from manufacturing considerations.

This stage involves the detailed stress analysis.

Fifth Step: The last stage in design process is to prepare the blue prints of assembly and individual component.
On these drawings, the material of the components, dimensions and tolerances, surface finish and machining methods are specified.
The designer prepare two separate lists of components

- Standard components to be purchased directly from the market;
- Special components to be machined in the factory;

Thus the machine design or mechanical design process is a systematic step-by-step approach from known specification to unknown solution.

Definition Related to Machine Design:

1. **Empirical Design:** The design based on previous experience and existing practice is known as empirical design. In empirical design various dimensions are set out as a proportion of certain main dimension. For example the various dimensions of rigid coupling are given as a proportion of shaft diameter.
2. **Rational Design:** It is purely a mathematical design based on principles of engg. mechanics, mechanics of solids, or simply of engineering science etc. In this design mechanics of solids is used to decide the different dimensions of a product. i.e By solving the basic design equations and using optimization techniques. For example the design of shaft diameter

3. **Combined & Rational Design:** In this design the components is designed on the basis of mechanics of solids but in no case the individual dimensions to be less than the existing or past practice. For example the design of hub diameter of protective type rigid flange coupling by using the combined and rational design.

4. **Design by Innovation:** “Innovation is the creative act where by an idea is conceived and brought into successful practice.” These days the competition has widened. There is no lace for slow pace technological changes. Present circumstances need faster, and bolder improvement or design. To achieve the faster and bolder design utilizes new innovations like CAD, CATIA, ANSYS etc. in the field of design. This is known as design by innovation, In this the technical risks are great.

5. **New Design:** “If the design is created from a scratch through the application of scientific principles, technical ability and creative thinking of the designer known as New Design or “Inventive or Creative Design”.” If a problem arises that suggests a machine for the solution and no suitable machine exists, a designer will have to create it then the machine will be termed as NEW DESIGN. For example the locomotives, airplane, steam engine were considered as New Design when they had come in shape first time for use.

6. **Adaptive Design & Redesign:** There are many fields where the development has practically cased e.g. bicycles, I.C. engines etc. So there is hardly anything left for the designer to do except make minor modifications usually in the dimensions of the product. This work is concerned with adaptation of existing design. The final outcome does not differ much from the initial product. Thus much of designer work consists of redesign.
   
   **For example:** A press is to be designed for more capacity, a turret lathe is to be designed to accommodate large stock, an automobile engine is to be designed to produce more power. For all these examples the mechanisms are already designed and the designer does not devise different mechanism. The designer only need to decide which part is to be changed and what part is to remain same to achieve the aim. But in making these changes the designer always keep in mind that the changes are costly hence the redesign must produce desired result with minimum alteration.
The mechanical design or machine design procedure may be shown schematically as follows:

**Figure 1.2**

**MORPHOLOGY OF DESIGN**: Morphology of design is the chronological vertical structure of the various phases or steps together from the engineering analysis to the retirement of the product. Thus morphology of design includes the following steps:

(i) **Feasibility Study**: The aim is to produce a number of feasible and useful solutions. Here the alternatives are assessed in stages. The first stage is made on the basis of common sense. Many of the broad solutions may not be worth consideration. Considering technical feasibility some of the solutions can be eliminated. The last stage is the economic assessment. Systematic technical, economic, social and legal considerations provide a rapid convergence towards the useful solutions.

(ii) **Preliminary Design**: Feasibility study yields a set of useful solutions. The aim in this phase is to choose the optimal solution. To do this, criterion of optimization must be explicitly delineated. The chosen alternative is then tested and predictions are made concerning its performance.

(iii) **Detailed Design**: The purpose of the detailed design is to produce a complete engineering description of a tested and producible design for manufacture. A detailed design includes manufacturing drawings with tolerances.
(iv) **Planning for Manufacturing**: A procedure sheet is to be made which contains a sequence of manufacturing operations that must be performed on the component. It specifies clearly the tooling, fixtures and production machines. This phase may include planning, and inventory control, quality control system, the fixing of standard time and labor cost for each operation.

(v) **Planning for Distribution, Use and Retirement of the Product**: The success of a design depends on the skill exercised in marketing the product. Also the user-oriented concern such as reliability, ease of maintenance, product safety, convenience in use, aesthetic appeal, economy and durability must met. The product life considering actual wear or deterioration, and technological obsolescence must be planned.

**Figure 1.3**

**Factors to be Considered in Machine Design:**
There are many factors to be considered while attacking a design problem. In many cases these are a common sense approach to solving a problem. Some of these factors are as follows:

(a) What device or mechanism to be used? This would decide the relative arrangement of the constituent elements.

(b) Material

(c) Forces on the elements

(d) Size, shape and space requirements. The final weight of the product is also a major concern.

(e) The method of manufacturing the components and their assembly.

(f) How will it operate?

(g) Reliability and safety aspects

(h) inspectibility

(i) Maintenance, cost and aesthetics of the designed product.
What device or mechanism to be used
This is best judged by understanding the problem thoroughly. Sometimes a particular function can be achieved by a number of means or by using different mechanisms and the designer has to decide which one is most effective under the circumstances. A rough design or layout diagram may be made to crystallize the thoughts regarding the relative arrangement of the elements.

Material
This is a very important aspect of any design. A wrong choice of material may lead to failure, over or undersized product or expensive items. The choice of materials is thus dependent on suitable properties of the material for each component, their suitability of fabrication or manufacture and the cost.

Load
The external loads cause internal stresses in the elements and these stresses must be determined accurately since these will be used in determining the component size. Loading may be due to:
i) Energy transmission by a machine member.
ii) Dead weight.
iii) Inertial forces.
iv) Thermal effects.
v) Frictional forces.

In other ways loads may be classified as:
   i) Static load- Does not change in magnitude and direction and normally increases gradually to a steady value.

   ii) Dynamic load-
       a) changes in magnitude- for e.g. traffic of varying weight passing a bridge.
       b) changes in direction- for e.g. load on piston rod of a double acting cylinder.

Vibration and shock loading are types of dynamic loading. The nature of these loads are shown in figure-1.4

![Figure 1.4, Types of Load](image)

Size, shape, space requirements and weight
Preliminary analysis would give an approximate size but if a standard element is to be chosen, the next larger size must be taken. Shapes of standard elements are known but for non-standard element, shapes and space requirements must depend on available space in a particular machine assembly. A scale layout drawing is often useful to arrive at an initial shape and size. Weight is important depending on application. For example, an aircraft must always be made light. This means that the material chosen must have the required strength yet it must be light. Similar arguments apply to choice of material for ships and there too light materials are to be chosen. Portable equipment must be made light.

**Manufacture**

Care must always be taken to ensure that the designed elements may be manufactured with ease, within the available facilities and at low cost.

**How will it operate?**

In the final stage of the design a designer must ensure that the machine may be operated with ease. In many power operated machines it is simply a matter of pressing a knob or switch to start the machine. However in many other cases, a sequence of operations is to be specified. This sequence must not be complicated and the operations should not require excessive force. Consider the starting, accelerating and stopping a scooter or a car. With time tested design considerations, the sequences have been made user-friendly and as in any other product, these products too go through continuous innovation and development.

**Reliability and safety**

Reliability is an important factor in any design. A designed machine should work effectively and reliably. The probability that an element or a machine will not fail in use is called reliability. Reliability lies between $0 \leq R < 1$. To ensure this, every detail should be examined. Possible overloading, wear of elements, excessive heat generation and other such detrimental factors must be avoided. There is no single answer for this but an overall safe design approach and care at every stage of design would result in a reliable machine.

Safety has become a matter of paramount importance these days in design. Machines must be designed to serve mankind, not to harm it. Industrial regulations ensure that the manufacturer is liable for any damage or harm arising out of a defective product. Use of a factor of safety only in design does not ensure its overall reliability.

**Maintenance, cost and aesthetics**

Maintenance and safety are often interlinked. Good maintenance ensures good running condition of machinery. Often a regular maintenance schedule is maintained and a thorough check up of moving and loaded parts is carried out to avoid catastrophic failures. Low friction and wear is maintained by proper lubrication. This is a major aspect of design since wherever there are moving parts, friction and wear are inevitable. High friction leads to increased loss of energy. Wear of machine parts leads to loss of material and premature failure. Cost and aesthetics are essential considerations for product design. Cost is
essentially related to the choice of materials which in turn depends on the stresses developed in a given condition. Although in many cases aesthetic considerations are not essential aspects of machine design, ergonomic aspects must be taken into considerations.

**Standards and Standardization:**

**Standards In Design:**
Standard is a set of specifications, defined by a certain body or an organization, to which various characteristics of a component, a system, or a product should conform. The characteristics may include: dimensions, shapes, tolerances, surface finish etc.

**Types Of Standards Used In Machine Design:**
Based on the defining bodies or organization, the standards used in the machine design can be divided into following three categories:

(i) **Company Standards:** These standards are defined or set by a company or a group of companies for their use.

(ii) **National Standards:** These standards are defined or set by a national apex body and are normally followed throughout the country. Like BIS, AWS.

(iii) **International Standards:** These standards are defined or set by international apex body and are normally followed throughout the world. Like ISO, IBWM.

**Advantages:**
- Reducing duplication of effort or overlap and combining resources
- Bridging of technology gaps and transferring technology
- Reducing conflict in regulations
- Facilitating commerce
- Stabilizing existing markets and allowing development of new markets
- Protecting from litigation

**Selection of Preferred Sizes**

French balloonist and Engineer Charles Renard suggested the method to specify the sizes of the product to satisfy the needs of the customers with the minimum number of sizes in the given range by using the G.P. He gave the five basic series having their specific series factor as follows:

<table>
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<th>SERIES</th>
<th>SERIES FACTOR</th>
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<tr>
<td>R-5</td>
<td>(5\sqrt{10})</td>
</tr>
<tr>
<td>R-10</td>
<td>(10\sqrt{10})</td>
</tr>
<tr>
<td>R-20</td>
<td>(20\sqrt{10})</td>
</tr>
<tr>
<td>R-40</td>
<td>(40\sqrt{10})</td>
</tr>
<tr>
<td>R-80</td>
<td>(80\sqrt{10})</td>
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The second size is obtained by multiplying the series factor to the first size. The third size is obtained by multiplying the series factor to the second size. And this procedure is repeated until the whole range is covered. Thus the sizes are specified in the given range and this is known as the selection of preferred sizes.

For the number of R-10 series the series factor will be \( \sqrt[10]{10} \).

**ADVANTAGES OF SELECTION OF PREFERRED SERIES:** The advantages of preferred series are as follows:

1. The differences in two successive terms have a fixed percentage.
2. Provides small steps for small quantities and large steps for the large quantities. It is in conformance with the mode of variation found in nature.
3. The product range is covered with minimum number of sizes without restricting the choices of the customers.
4. It helps the designer to avoid the selection of sizes arbitrarily.

Q. It is required to standardize load carrying capacities of dumpers in a manufacturing unit. The maximum and minimum capacities of such dumpers are 630 and 40 kN respectively. The company is interested in developing seven models in this range. Specify their load carrying capacities.

**Sol.:** Let the first Model capacity = 40 kN;

Using the R-5 series having series factor 1.58,

First Model = 40 kN;

Second Model = 40 \times 1.58 = 63.2 \approx 63 kN;

Third Model = 63.2 \times 1.58 = 99.856 \approx 100 kN;

Fourth Model = 99.856 \times 1.58 = 157.77248 \approx 158 kN;

Fifth Model = 157.77248 \times 1.58 = 249.2805 \approx 249 kN;

Sixth Model = 249.2805 \times 1.58 = 393.86 \approx 394 kN;

Seventh Model = 393.86 \times 1.58 = 622.3038 \approx 622 kN;

**B.I.S DESIGNATIONS OF THE PLAIN CARBON STEEL:** Plain carbon steel is designated according to BIS as follows:

1. The first one or two digits indicate the 100 times of the average percentage content of carbon.
2. Followed by letter “C”
3. Followed by digits indicates 10 times the average percentage content of Manganese “Mn”.

**B.I.S DESIGNATIONS OF ALLOY STEEL:** Alloy carbon steel is designated according to BIS as follows:

1. The first one or two digits indicate the 100 times of the average percentage content of carbon.
2. Followed by the chemical symbol of chief alloying element.
3. Followed by the rounded off the average percentage content of alloying element as per international standards.
4. Followed by the chemical symbol of alloying elements followed by their average percentage content rounded off as per international standards in the descending order.

5. If the average percentage content of any alloying element is less than 1%, it should be written with the digits up to two decimal places and underlined.

   (i) 45C8 = Plain carbon steel having average %age of carbon of 0.45% and 0.8% Mn.
   (ii) Fe250 = Grey cast iron having minimum tensile strength of 250 N/mm²;
   (iii) XT30Ni4Cr2V65 = High speed tool steel having
         Average %age content of “C” = 0.30%;
         Average %age content of “Ni” = 4.0%;
         Average %age content of “Cr” = 2.0%;
         Average %age content of “V” = 0.65%;
   (iv) 37Mn2 = Alloy Steel having
         Average %age content of “C” = 0.37%;
         Average %age content of “Mn” = 2.0%;

BASIC CRITERION FOR SELECTION OF MATERIAL:

The basic criterions considered by a designer for the selection of a material for a particular application are:

- Availability of material;
- Cost of material;
- Manufacturing Considerations;
- Material Properties.

Availability of Material:

The material which is available readily and in abundance in the market should be selected.

Cost of Material:

The material should be selected such that the total cost should be minimum and within the specified limits. The total cost includes the cost of material and cost of processing of the material.

Manufacturing Considerations:

The material should be suitable for the required manufacturing processes to make the product or component.

Material Properties:
The material possesses the required mechanical, physical and chemical properties required for the application.

CLASSIFICATION OF ENGINEERING MATERIALS:

![Diagram of classification of materials]

**Ferrous Materials:**

**Cast iron:**

It is an alloy of iron, carbon and silicon and it is hard and brittle. Carbon content may be within 1.7% to 3% and carbon may be present as free carbon or iron carbide Fe₃C. In general, the types of cast iron are (a) grey cast iron and (b) white cast iron (c) malleable cast iron (d) spheroidal or nodular cast iron (e) austenitic cast iron (f) abrasion-resistant cast iron.

(a) **Grey cast iron:** Carbon here is mainly in the form of graphite. This type of cast iron is inexpensive and has high compressive strength. Graphite is an excellent solid lubricant and this makes it easily machinable but brittle. Some examples of this type of cast iron are FG20, FG35 or FG35Si15.

(b) **White cast iron:** In these cast irons carbon is present in the form of iron carbide (Fe₃C) which is hard and brittle. The presence of iron carbide increases hardness and makes it difficult to machine. Consequently, these cast irons are abrasion resistant.

(c) **Malleable cast iron:** These are white cast irons rendered malleable by annealing. These are tougher than grey cast iron and they can be twisted or bent without fracture. They have excellent machining properties and are inexpensive. Malleable cast iron are used for making parts where forging is expensive such as hubs for wagon wheels, brake supports.

Figure 1.5 Classification of Materials
(d) **Spheroidal or nodular graphite cast iron**: In these cast irons graphite is present in the form of spheres or nodules. They have high tensile strength and good elongation properties.

(e) **Austenitic cast iron**: Depending on the form of graphite present these cast iron can be classified broadly under two headings:

- Austenitic flake graphite iron;
- Austenitic spheroidal or nodular graphite iron

These are alloy cast irons and they contain small percentages of silicon, manganese, sulphur, phosphorus etc. They may be produced by adding alloying elements viz. nickel, chromium, molybdenum, copper and manganese in sufficient quantities. These elements give more strength and improved properties. They are used for making automobile parts such as cylinders, pistons, piston rings, brake drums etc.

(f) **Abrasion resistant cast iron**: These are alloy cast iron and the alloying elements render abrasion resistance.

**Wrought iron**: This is a very pure iron where the iron content is of the order of 99.5%. It is produced by re-melting pig iron and some small amount of silicon, sulphur, or phosphorus may be present. It is tough, malleable and ductile and can easily be forged or welded. It cannot however take sudden shock. Chains, crane hooks, railway couplings and such other components may be made of this iron.

**Steel**: This is by far the most important engineering material and there is an enormous variety of steel to meet the wide variety of engineering requirements. Steel is basically an alloy of iron and carbon in which the carbon content can be less than 1.7% and carbon is present in the form of iron carbide to impart hardness and strength. Two main categories of steel are (a) Plain carbon steel and (b) alloy steel.

(a) **Plain carbon steel**: The properties of plain carbon steel depend mainly on the carbon percentages and other alloying elements are not usually present in more than 0.5 to 1% such as 0.5% Si or 1% Mn etc. The plain carbon steel further may be classified in the following categories:

- Dead mild steel- upto 0.15% C
- Low carbon steel or mild steel- 0.15 to 0.46% C
- Medium carbon steel- 0.45 to 0.8% C.
- High carbon steel- 0.8 to 1.5% C

Usually in these steels in general higher carbon percentage indicates higher strength.

(b) **Alloy steel**: these are steels in which elements other than carbon are added in sufficient quantities to impart desired properties, such as wear resistance, corrosion resistance, electric or magnetic properties. Chief alloying elements added are usually nickel for strength and toughness, chromium for hardness and strength, tungsten for hardness at
elevated temperature, vanadium for tensile strength, manganese for high strength in hot rolled and heat treated condition, silicon for high elastic limit, cobalt for hardness and molybdenum for extra tensile strength.

**Non-Ferrous Metals:**

Metals containing elements other than iron as their chief constituents are usually referred to as non-ferrous metals. There is a wide variety of non-metals in practice. However, only a few exemplary ones are discussed below:

**Aluminium** - This is the white metal produced from Alumina. In its pure state it is weak and soft but addition of small amounts of Cu, Mn, Si and Magnesium makes it hard and strong. It is also corrosion resistant, low weight and non-toxic.

**Duralumin** - This is an alloy of 4% Cu, 0.5% Mn, 0.5% Mg and aluminium. It is widely used in automobile and aircraft components.

**Y-alloy** - This is an alloy of 4% Cu, 1.5% Mn, 2% Ni, 6% Si, Mg, Fe and the rest is Al. It gives large strength at high temperature. It is used for aircraft engine parts such as cylinder heads, piston etc.

**Magnalium** - This is an aluminium alloy with 2 to 10% magnesium. It also contains 1.75% Cu. Due to its light weight and good strength it is used for aircraft and automobile components.

**Copper alloys:**

Copper is one of the most widely used non-ferrous metals in industry. It is soft, malleable and ductile and is a good conductor of heat and electricity. The following two important copper alloys are widely used in practice:

**Brass (Cu-Zn alloy):**

It is fundamentally a binary alloy with Zn up to 50%. As Zn percentage increases, ductility increases up to ~37% of Zn beyond which the ductility falls. This is shown in figure-1.6. Small amount of other elements viz. lead or tin imparts other properties to brass. Lead gives good machining quality and tin imparts strength. Brass is highly corrosion resistant, easily machinable and therefore a good bearing material.

![Figure 1.6, Variation of ductility of brass with percentage of zinc.](image)
Non-Metals
Non-metallic materials are also used in engineering practice due to principally their low cost, flexibility and resistance to heat and electricity. Though there are many suitable non-metals, the following are important few from design point of view:

Timber- This is a relatively low cost material and a bad conductor of heat and electricity. It has also good elastic and frictional properties and is widely used in foundry patterns and as water lubricated bearings.

Leather- This is widely used in engineering for its flexibility and wear resistance. It is widely used for belt drives, washers and such other applications.

Rubber- It has high bulk modulus and is used for drive elements, sealing, vibration isolation and similar applications.

Plastics: These are synthetic materials which can be moulded into desired shapes under pressure with or without application of heat. These are now extensively used in various industrial applications for their corrosion resistance, dimensional stability and relatively low cost. There are two main types of plastics:

(a) Thermosetting plastics- Thermostatic plastics are formed under heat and pressure. It initially softens and with increasing heat and pressure, polymerisation takes place. This results in hardening of the material. These plastics cannot be deformed or remoulded again under heat and pressure. Some examples of thermosetting plastics are phenol formaldehyde (Bakelite), phenol-furfural (Durite), epoxy resins, phenolic resins etc.

(b) Thermoplastics- Thermoplastics do not become hard with the application of heat and pressure and no chemical change takes place. They remain soft at elevated temperatures until they are hardened by cooling. These can be re-melted and remoulded by application of heat and pressure. Some examples of thermoplastics are cellulose nitrate (celluloid), polythene, polyvinyl acetate, polyvinyl chloride (PVC) etc.

Mechanical Properties of Common Engineering Materials:

The important properties from design point of view are:

(a) Elasticity- This is the property of a material to regain its original shape after deformation when the external forces are removed. All materials are plastic to some extent but the degree varies, for example, both mild steel and rubber are elastic materials but steel is more elastic than rubber.
(b) **Plasticity**-
This is associated with the permanent deformation of material when the stress level exceeds the yield point. Under plastic conditions materials ideally deform without any increase in stress. A typical stress-strain diagram for an elastic-perfectly plastic material is shown in the figure-1.7. A typical example of plastic flow is the indentation test where a spherical ball is pressed in a semi-infinite body.

![Stress-strain diagram](image)

**Figure 1.7, Stress-strain diagram of an elastic-perfectly plastic material and the plastic indentation.**

(c) **Hardness**-
Property of the material that enables it to resist permanent deformation, penetration, indentation etc. Size of indentations by various types of indenters are the measure of hardness e.g. Brinnel hardness test, Rockwell hardness test, Vickers hardness (diamond pyramid) test. These tests give hardness numbers which are related to yield pressure (MPa).

(d) **Ductility**-
This is the property of the material that enables it to be drawn out or elongated to an appreciable extent before rupture occurs. The percentage elongation or percentage reduction in area before rupture of a test specimen is the measure of ductility. Normally if percentage elongation exceeds 15% the material is ductile and if it is less than 5% the material is brittle. Lead, copper, aluminium, mild steel are typical ductile materials.

(e) **Malleability**-
It is a special case of ductility where it can be rolled into thin sheets but it is not necessary to be so strong. Lead, soft steel, wrought iron, copper and aluminium are some materials in order of diminishing malleability.

(f) **Brittleness**-
This is opposite to ductility. Brittle materials show little deformation before fracture and failure occur suddenly without any warning. Normally if the elongation is less than 5% the material is considered to be brittle. e.g. cast iron, glass, ceramics are typical brittle materials.
(g) **Resilience**-

This is the property of the material that enables it to resist shock and impact by storing energy. The measure of resilience is the strain energy absorbed per unit volume. For a rod of length \( L \) subjected to tensile load \( P \), a linear load-deflection plot is shown in figure-1.8.

\[
\text{Strain energy stored} = \frac{1}{2} P \times \delta L = \frac{1}{2} \frac{P \times \delta L}{A \times L} \times A \times L = \frac{1}{2} \sigma \varepsilon \times V
\]

**Figure 1.8, A linear load-deflection curve**

(h) **Toughness**-

This is the property which enables a material to be twisted, bent or stretched under impact load or high stress before rupture. It may be considered to be the ability of the material to absorb energy in the plastic zone. The measure of toughness is the amount of energy absorbed after being stressed upto the point of fracture.

(i) **Creep**-

When a member is subjected to a constant load over a long period of time it undergoes a slow permanent deformation and this is termed as “creep”. This is dependent on temperature. Usually at elevated temperatures creep is high.
Design Against Static Load:

Modes Of Failure Of Materials Subjected to Static Load: A mechanical component may fail, that is, it may be unable to perform its function satisfactorily, as a result of any one of three modes of failure:

i. Failure by elastic deflection;
ii. Failure by general yielding
iii. Failure by fracture;

(i) **Failure by Elastic Deflection:** In applications like transmission shafts supporting gears, the maximum force acting on the shaft without affecting its performance is limited by the permissible elastic deflection. Lateral or torsional rigidity is considered as the criterion of design in such cases. Sometimes the elastic deflection results in unstable conditions, such as buckling of columns or vibrations and may cause deterioration in performance of the mechanical system and it is considered the case of failure of the design due to the elastic deflection. The moduli of elasticity and rigidity are the important properties and the dimensions of the component are determined by the load deflection equation.

(ii) **Failure by General Yielding:** A mechanical component made of ductile material loses its engineering usefulness due to a large amount of plastic deformation after the yield point stress is reached. Considerable portion of the component is subjected to plastic deformation called general yielding. The yield strength of the material is an important property when a component is designed against general yielding.

(iii) **Failure by Fracture:** Components made of brittle material ceases to function satisfactorily because of the sudden fracture without any plastic deformation. The failure in this case is sudden and total. In such cases, ultimate tensile strength of the material is an important property for determining the dimensions of these components.

**Factor of Safety:**

Determination of stresses in structural or machine components would be meaningless unless they are compared with the material strength. If the induced stress is less than or equal to the limiting material strength then the designed component may be considered to be safe and an indication about the size of the component is obtained. The strength of various materials for engineering applications is determined in the laboratory with standard specimens. For example, for tension and compression tests a round rod of specified dimension is used in a tensile test machine where load is applied until fracture occurs. This test is usually carried out in a Universal testing machine. Similar tests are carried out for bending, shear and torsion and the results for different materials are available in handbooks. For design purpose an allowable stress is used in place of the critical stress to take into account the uncertainties including the following:
1) Uncertainty in loading.
2) Inhomogeneity of materials.
3) Various material behaviours, e.g. corrosion, plastic flow, creep.
4) Residual stresses due to different manufacturing process.
5) Fluctuating load (fatigue loading): Experimental results and plot- ultimate strength depends on number of cycles.
6) Safety and reliability.
7) Consequences of failure – human safety and economy
8) Cost of providing a larger factor of safety.

For ductile materials, the yield strength and for brittle materials the ultimate strengths are taken as the critical stress. An allowable stress is set considerably lower than the ultimate strength. The ratio of ultimate to allowable load or stress is known as factor of safety i.e. The factor of safety can be defined as the ratio of the material strength or failure stress to the allowable or working stress. Mathematically it may be expressed as

\[
f.o.s = \frac{S_{yt}}{S_{yt\text{ working or allowable stress}}} \text{; for ductile materials,}
\]

\[
f.o.s = \frac{S_{ut}}{S_{ut\text{ working or allowable stress}}} \text{; for brittle materials,}
\]

\[
f.o.s = \frac{S_{e}}{S_{e\text{ working or allowable stress}}} \text{; for fatigue loading of materials,}
\]

Symbols are having the usual meanings.

The factor of safety must be always greater than unity. It is easier to refer to the ratio of stresses since this applies to material properties.

**Principal Stresses:**

Figure 1.9(a) Transformation of stresses from x-y to x'-y' co-ordinate system

Figure 1.9(b) Stresses on an isolated triangular element
Consider a state of general plane stress in x-y co-ordinate system. Transforming it to another stress system in, say, x'-y' co-ordinates, that is inclined at an angle θ, as shown in figure 1.9(a). A two dimensional stress field acting on the faces of a cubic element is shown in figure 1.9(b). In plane stress assumptions, the non-zero stresses are \( \sigma_x, \sigma_y \) and \( \tau_{xy} = \tau_{yx} \). The stresses \( \sigma'_x \) and \( \tau'_{xy} \) are induced on the plane AC is inclined at an angle \( \theta \) in the isolated element ABC as shown in figure 1.9(b).

Considering the force equilibrium in x-direction we may write
\[
\sigma'_x = \sigma_x \cos \theta + \sigma_y \sin \theta + \tau_{xy} \sin \theta \cos \theta
\]

This may be reduced to
\[
(1.1) \quad \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta = 0
\]

Similarly, force equilibrium in y-direction gives
\[
(1.2) \quad \frac{\sigma_y - \sigma_x}{2} \sin 2\theta + \tau_{xy} \cos 2\theta = 0
\]

Since plane AC can assume any arbitrary inclination, a stationary value of \( \sigma_x' \) is given by
\[
\frac{d\sigma'_x}{d\theta} = 0
\]

This gives
\[
tan 2\theta = \frac{2 \tau_{xy}}{\sigma_x - \sigma_y} \quad (1.3)
\]

This equation has two roots and let the two values of \( \theta \) be \( \theta_1 \) and \( (\theta_1 + 90^\circ) \). Therefore these two planes are the planes of maximum and minimum normal stresses. Now if we set \( \tau'_{xy} = 0 \) we get the values of \( \theta \) corresponding to planes of zero shear stress. This also gives
\[
tan 2\theta = \frac{2 \tau_{xy}}{\sigma_x - \sigma_y} \quad (1.4)
\]

And equation (1.3) and (1.4) are same, indicating that at the planes of maximum and minimum stresses no shearing stress occurs. These planes are known as Principal planes and stresses acting on these planes are known as Principal stresses. From equation (1.1) and (1.3) the principal stresses are given as
\[
\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left( \frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2} \quad (1.5)
\]

In the same way, condition for maximum shear stress is obtained from
\[
\frac{d\tau'_{xy}}{d\theta} = 0
\]

This gives
\[ \tan 2\theta = \frac{\sigma_x - \sigma_y}{2\tau_{xy}} \]  \hspace{1cm} \text{(1.6)}

This also gives two values of \( \theta \) say \( \theta_2 \) and \( (\theta_2 + 90^\circ) \), at which shear stress is maximum or minimum. Combining equations (1.2) and (1.6) the two values of maximum shear stresses are given by

\[ \tau_{\text{max}} = \pm \sqrt{\left( \frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2} \]  \hspace{1cm} \text{(1.7)}

One important thing to note here is that values of \( \tan 2\theta_2 \) is negative reciprocal of \( \tan 2\theta_1 \) and thus \( \theta_1 \) and \( \theta_2 \) are \( 45^\circ \) apart. This means that principal planes and planes of maximum shear stresses are \( 45^\circ \) apart. It also follows that although no shear stress exists at the principal planes, normal stresses may act at the planes of maximum shear stresses.

Q. Consider an element with the following stress system shown in figure 1.10, \( \sigma_x = -10 \text{ MPa}, \sigma_y = +20 \text{ MPa}, \tau = -20 \text{ MPa} \). Find the principal stresses and show their senses on a properly oriented element.

\[ \text{Solution:} \]

The principal stresses are

\[ \sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left( \frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2} \]

\[ \sigma_{1,2} = \frac{-10 + 20}{2} \pm \sqrt{\left( \frac{-10 - 20}{2} \right)^2 + (-20)^2} \]

\[ \sigma_{1,2} = 5 \pm 25; \]

\[ \sigma_1 = +30 \text{ MPa}; \]

\[ \sigma_{1,2} = -20 \text{ MPa} \]

This gives -20 MPa and 30 MPa, the principal planes are given by

\[ \tan 2\theta = \frac{2(-20)}{-10 - 20} = 1.33 \]

\[ 2\theta = \tan^{-1}(1.33) = 53.130^\circ; \]

\[ \theta = 26.56^\circ \]

\[ \therefore \theta_1 = 26.56^\circ \& \]

\[ \theta_2 = 26.56^\circ + 90^\circ = 116.56^\circ \]

Thus the oriented element to show the principal stresses is shown in figure 1.11.
Stresses in Bending and Torsion:

**Bending stresses:** Consider two sections ab and cd in a beam subjected to a pure bending. Due to bending the top layer is under compression and the bottom layer is under tension. This is shown in figure-1.12. This means that in between the two extreme layers there must be a layer which remains un-stretched and this layer is known as neutral layer. Let this be denoted by NN'.

![Figure 1.11, Orientation of the loaded element in the left to show the principal stresses.](image1)

![Figure 1.12, Pure bending of beams](image2)

We consider that a plane section remains plane after bending- a basic assumption in pure bending theory. If the rotation of cd with respect to ab is $d\phi$ the contraction of a layer $y$ distance away from the neutral axis is given by $ds = y \, d\phi$ and original length of the layer is $x = R \, d\phi$, $R$ being the radius of curvature of the beam. This gives the strain $\varepsilon$ in the layer as...
\[ \varepsilon = \frac{y}{R} \]  ----------- (1.8) \]

We also consider that the material obeys Hooke’s law \( \sigma = E\varepsilon \). This is another basic assumption in pure bending theory and substituting the expression for \( \varepsilon \) we have

\[ \frac{\sigma}{E} = \frac{y}{R} \]  ----------- (1.9) \]

Consider now a small element \( dA \) y distance away from the neutral axis. This is shown in the figure 1.13.

\[
\text{Axial force on the element} \\
dF_x = \sigma_x \times dA
\]  ----------- (1.10) \]

And considering the linearity in stress variation across the section

\[ \frac{\sigma_x}{\sigma_{\text{max}}} = \frac{y}{d} \]  ----------- (1.11) \]

we have where \( \sigma_x \) and \( \sigma_{\text{max}} \) are the stresses at distances \( y \) and \( d \) respectively from the neutral axis.

The axial force on the element is thus given by equation (1.10) and (1.11)

\[ dF_x = \frac{y \times \sigma_{\text{max}}}{d} \times dA \]  ----------- (1.12) \]

For static equilibrium total force at any cross-section

\[ F = \int_{A} dF_x = \int_{A} \frac{y \times \sigma_{\text{max}}}{d} dA = 0 \]  ----------- (1.13) \]

This gives

\[ \int y \, dA = \bar{y} A = 0 \]  ----------- (1.14) \]

and since \( A \neq 0 \). This means that the neutral axis passes through the centroid. Again for static equilibrium total moment about NA must the applied moment \( M \). This is given by

\[
\text{Figure 1.13, Bending stress developed at any cross-section}
\]
\[ \frac{y \times \sigma_{\text{max}}}{d} \times y \times dA = M \quad \text{--- (1.15) ---} \]

It gives
\[ \sigma_{\text{max}} = \frac{M \times d}{J} \quad \text{--- (1.16) ---} \]

For any fibre at a distance of \( y \) from the centre line we may therefore write
\[ \sigma_{\text{max}} = \frac{M \times y}{I} \quad \text{--- (1.17) ---} \]

### Torsion of Circular Members:

A torque applied to a member causes shear stress. In order to establish a relation between the torque and shear stress developed in a circular member, the following assumptions are needed:

1. Material is homogeneous and isotropic
2. A plane section perpendicular to the axis of the circular member remains plane even after twisting i.e. no warping.

If the member is subjected to torque “\( T \)”, the maximum shear stress induced in the circular member of the radius “\( r \)” may be given as
\[ \tau = \frac{T \times r}{J}; \]
\( J \) = polar moment of inertia.

Thus the shear stress may be given as
\[ \tau = \frac{16 \times T}{\pi \times d^3}; \]
\[ J = \frac{\pi \times d^4}{32}; \text{ and } r = \frac{d}{2} \]

### Theories of Failure

**Necessity Of Having More Than One Theories Of Failure:** When a machine component is subjected to a uniaxial stress. It is easy to predict the failure because the stress and the strength can be compared directly. There is only one value of stress and one value of strength, be it yield strength, ultimate strength or shear strength and the method becomes simple. But the problem of predicting the failure of a component subjected to biaxial or triaxial stresses is more complicated.

This is because there are multiple stresses, but still only one significant strength. The different theories of failure have been proposed to predict the failure of the components subjected to biaxial or triaxial stresses and a shear stress.

When a machine element is subjected to a system of complex stress system, it is important to predict the mode of failure so that the design methodology may be based on a particular failure criterion. Theories of failure are essentially a set of failure criteria developed for the
Ease of design. In machine design an element is said to have failed if it ceases to perform its function. There are basically two types of mechanical failure:

(a) **Yielding** - This is due to excessive inelastic deformation rendering the machine part unsuitable to perform its function. This mostly occurs in ductile materials.

(b) **Fracture** - In this case the component tears apart in two or more parts. This mostly occurs in brittle materials. There is no sharp line of demarcation between ductile and brittle materials. However a rough guideline is that if percentage elongation is less than 5% then the material may be treated as brittle and if it is more than 15% then the material is ductile. However, there are many instances when a ductile material may fail by fracture. This may occur if a material is subjected to

(a) Cyclic loading.
(b) Long term static loading at elevated temperature.
(c) Impact loading.
(d) Work hardening.
(e) Severe quenching.

**Maximum principal stress theory (Rankine theory)**

According to this, if one of the principal stresses $\sigma_1$ (maximum principal stress), $\sigma_2$ (minimum principal stress) or $\sigma_3$ exceeds the yield stress, yielding would occur. In a two dimensional loading situation for a ductile material where tensile and compressive yield stress are nearly of same magnitude

$$\sigma_1 = \pm \sigma_y$$

$$\sigma_2 = \pm \sigma_y$$

Using this, a yield surface may be drawn, as shown in figure-1.14. Yielding occurs when the state of stress is at the boundary of the rectangle. Consider, for example, the state of stress of a thin walled pressure vessel. Here $\sigma_1 = 2\sigma_2$, $\sigma_1$ being the circumferential or hoop stress and $\sigma_2$ the axial stress. As the pressure in the vessel increases the stress follows the dotted line. At a point (say) a, the stresses are still within the elastic limit but at b, $\sigma_1$ reaches $\sigma_y$ although $\sigma_2$ is still less than $\sigma_y$. Yielding will then begin at point b. This theory of yielding has very poor agreement with experiment. However, the theory has been used successfully for brittle materials.

![Figure 1.14. Yield surface corresponding to maximum principal stress](image-url)
Maximum principal strain theory (St. Venant’s theory):
According to this theory, yielding will occur when the maximum principal strain just exceeds the strain at the tensile yield point in either simple tension or compression. If $\varepsilon_1$ and $\varepsilon_2$ are maximum and minimum principal strains corresponding to $\sigma_1$ and $\sigma_2$, in the limiting case

$$
\varepsilon_1 = \frac{1}{E} \left( \sigma_1 - \nu \sigma_2 \right) \quad |\sigma_1| \geq |\sigma_2|
$$

$$
\varepsilon_2 = \frac{1}{E} \left( \sigma_2 - \nu \sigma_1 \right) \quad |\sigma_2| \geq |\sigma_1|
$$

This gives

$$E\varepsilon_1 = \sigma_1 - \nu \sigma_2 = \pm \sigma_0$$

The boundary of a yield surface in this case is thus given as shown in $E\varepsilon_2 = \sigma_2 - \nu \sigma_1 = \pm \sigma_0$ figure 1.15

![Yield surface corresponding to maximum principal strain theory](image)

Figure 1.15, Yield surface corresponding to maximum principal strain theory

Maximum shear stress theory (Tresca theory):
According to this theory, yielding would occur when the maximum shear stress just exceeds the shear stress at the tensile yield point. At the tensile yield point $\sigma_2 = \sigma_3 = 0$ and thus maximum shear stress is $\sigma_y/2$. This gives us six conditions for a three-dimensional stress situation:

$$\sigma_1 - \sigma_2 = \pm \sigma_y$$

$$\sigma_2 - \sigma_3 = \pm \sigma_y$$

$$\sigma_3 - \sigma_1 = \pm \sigma_y$$
In a biaxial stress situation (figure-1.14) case, $\sigma_3 = 0$ and this gives

$$\sigma_1 - \sigma_2 = \sigma_y \quad \text{if } \sigma_1 > 0, \sigma_2 < 0$$
$$\sigma_1 - \sigma_2 = -\sigma_y \quad \text{if } \sigma_1 < 0, \sigma_2 > 0$$
$$\sigma_2 = \sigma_y \quad \text{if } \sigma_2 > \sigma_1 > 0$$
$$\sigma_1 = -\sigma_y \quad \text{if } \sigma_1 < \sigma_2 < 0$$
$$\sigma_1 = -\sigma_y \quad \text{if } \sigma_1 > \sigma_2 > 0$$
$$\sigma_2 = -\sigma_y \quad \text{if } \sigma_2 < \sigma_1 < 0$$

This criterion agrees well with experiment. In the case of pure shear, $\sigma_1 = -\sigma_2 = k$ (say), $\sigma_3 = 0$
and this gives $\sigma_1 - \sigma_2 = 2k = \sigma_y$. This indicates that yield stress in pure shear is half the
tensile yield stress and this is also seen in the Mohr’s circle (figure-1.17) for pure shear.

![Mohr's circle for pure shear](image)

**Figure 1.17, Mohr’s circle for pure shear**

**Maximum strain energy theory (Beltrami’s theory):**
According to this theory failure would occur when the total strain energy absorbed at a point per unit volume exceeds the strain energy absorbed per unit volume at the tensile yield point. This may be given

$$\frac{1}{2}(\sigma_1 \varepsilon_1 + \sigma_2 \varepsilon_2 + \sigma_3 \varepsilon_3) = \frac{1}{2} \sigma_y \varepsilon_y$$
$$\frac{1}{2}(\sigma_1 \varepsilon_1 + \sigma_2 \varepsilon_2 + \sigma_3 \varepsilon_3) = \frac{1}{2} \sigma_y \varepsilon_y$$

Substituting, $\varepsilon_1, \varepsilon_2, \varepsilon_3$ and $\varepsilon_y$ in terms of stresses we have

$$\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1 \sigma_2 + \sigma_2 \sigma_3 + \sigma_3 \sigma_1) = \sigma_y^2$$
This may be written as
\[ \left( \frac{\sigma_1}{\sigma_y} \right)^2 + \left( \frac{\sigma_2}{\sigma_y} \right)^2 - 2\nu \left( \frac{\sigma_1 \sigma_2}{\sigma_y^2} \right) = 1 \]

This is the equation of an ellipse and the yield surface is shown in figure-1.18

**Figure 1.18, Yield surface corresponding to Maximum strain energy theory**

It has been shown earlier that only distortion energy can cause yielding but in the above expression at sufficiently high hydrostatic pressure \( \sigma_1 = \sigma_2 = \sigma_3 = \sigma \) (say), yielding may also occur. From the above we may write

\[ \sigma^2 (3 - 2\nu) = \sigma_y^2 \]

And if \( \nu \sim 0.3 \), at stress level lower than yield stress, yielding would occur. This is in contrast to the experimental as well as analytical conclusion and the theory is not appropriate.

**Distortion energy theory (Von Mises yield criterion):**
According to this theory yielding would occur when total distortion energy absorbed per unit volume due to applied loads exceeds the distortion energy absorbed per unit volume at the tensile yield point. Total strain energy \( E_T \) and strain energy for volume change \( E_V \) can be given as

\[ E_T = \frac{1}{2} (\sigma_1 \varepsilon_1 + \sigma_2 \varepsilon_2 + \sigma_3 \varepsilon_3) \]
\[ E_V = \frac{3}{2} \sigma_{av} \varepsilon_{av} \]

Substituting strains in terms of stresses the distortion energy can be given as

\[ E_d = E_T - E_V = \frac{2(1 + \nu)}{6E} \left( \sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_3\sigma_1 \right) \]

At the tensile yield point, \( \sigma_1 = \sigma_y \), \( \sigma_2 = \sigma_3 = 0 \) which gives

\[ E_{dy} = \frac{2(1 + \nu)}{6E} \sigma_y^2 \]
The failure criterion is thus obtained by equating $E_d$ and $E_{dy}$, which gives
\[
\left(\sigma_1 - \sigma_2\right)^2 + \left(\sigma_2 - \sigma_3\right)^2 + \left(\sigma_3 - \sigma_1\right)^2 = 2\sigma_y^2
\]
In a 2-D situation if $\sigma_3 = 0$, the criterion reduces to
\[
\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 = \sigma_y^2
\]
i.e. \[
\left(\frac{\sigma_1}{\sigma_y}\right)^2 + \left(\frac{\sigma_2}{\sigma_y}\right)^2 - \left(\frac{\sigma_1}{\sigma_y}\right)\left(\frac{\sigma_2}{\sigma_y}\right) = 1
\]
This is an equation of ellipse and the yield surface is shown in figure-1.19. This theory agrees very well with experimental results and is widely used for ductile materials.

![Figure 1.19, Yield surface corresponding to von Mises yield criterion](image1)

**Superposition of Yield Surface:**
A comparison among the different failure theories can be made by superposing the yield surfaces as shown in figure- 1.20

![Figure 1.20, Comparison of different failure theories](image2)
It is clear that an immediate assessment of failure probability can be made just by plotting any experimental in the combined yield surface. Failure of ductile materials is most accurately governed by the distortion energy theory where as the maximum principal strain theory is used for brittle materials.

Q. A beam of uniform cross section is fixed at one end and carries an electric motor weighing 400 N at a distance of 300 mm from the fixed end. The maximum bending stress in the beam is 40 MPa. Find the width and depth of the beam if the depth is twice that of the width.

SOLUTION:

Referring the figure given in the problem
b = Width of the beam in mm;
h = Depth of the beam in mm

\[ Z \text{(section modulus)} = \frac{bh^2}{6} = \frac{b(2b)^2}{6} = \frac{2b^3}{3} \text{ mm}^3; \]

At the fixed end, the maximum bending moment will be,
M = WL = 400 \times 300 = 120000 \text{ N - mm};
Bending moment \( \sigma_b \) is given as 40 MPa;

\[ 40 = \frac{M}{Z} = \frac{120000 \times 3}{2b^3} \]

b = 16.5 mm;
h = 2b = 2 \times 16.5 = 33 mm;

Q. A bolt is subjected to an axial pull of 10 kN and transverse shear force of 5 kN. The yield strength of the bolt material is 300 MPa. Considering the factor of safety of 2. Determine the diameter of the shaft using (i) maximum shear stress theory and (ii) distortion energy theory.

Solution: Assuming the problem for designing the bolt diameter as this problem started with the data for bolt but asked to find the diameter of the shaft.

Given: Transverse load “P_t” = 10 kN = 1000 N; Shear Load “P_s” = 5 kN =5000 N; The yield strength of the material “S_yt” = 300 MPa; Factor of safety “f.o.s.” = 2.0;

The permissible stress: \[ \sigma_d = \frac{S_y}{f.o.s} = \frac{300}{2.0} = 150 \text{ MPa}; \]

\[ \text{--------------(1)} \]
The axial stress:  \[ \sigma_a = \frac{P_t}{\pi d^2/4} = \frac{4 \times 10000}{2} = \frac{12732.395}{4} \text{ MPa}; \text{----------(2)} \]

The transverse shear stress:
\[ \tau = \frac{P_t}{\pi d^2/4} = \frac{4 \times 5000}{4} = \frac{6366.1977}{4} \text{ MPa}; \text{----------(3)} \]

The principal stresses and maximum shear stress:
\[ \sigma_{1,2} = \frac{\sigma_a}{2} \pm \sqrt{\left(\frac{\sigma_a}{2}\right)^2 + \tau^2} = \frac{12732.395}{2} \pm \sqrt{\left(\frac{12732.395}{2}\right)^2 + \left(\frac{6366.1977}{2}\right)^2} = \frac{6366.1975 \pm 9003.163}{4} \text{ MPa}; \text{----------(4)} \]
\[ \sigma_1 = \frac{15369.3605}{d^2} \text{ MPa}; \sigma_2 = -\frac{2636.9655}{d^2} \text{ MPa}; \text{-----------(4)} \]
\[ \tau_{\text{max}} = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{2} \left[ \frac{15369.3605}{d^2} + \frac{2636.9655}{d^2} \right] = \frac{9003.163}{d^2} \text{-----------(5)} \]

(i) According to maximum shear stress theory:
\[ \tau_{\text{max}} \leq \frac{S_{yt}}{2 \times f.o.s} \]
\[ \frac{9003.163}{d^2} \leq \frac{300}{2 \times 2} \]
\[ d \geq \sqrt{\frac{4 \times 9003.163}{300}} = 10.9564 \text{ mm}; \]

(ii) According to distortion energy theory:
\[ \sigma_1^2 + \sigma_2^2 - 2\sigma_1 \times \sigma_2 \leq \left( \frac{S_{yt}}{f.o.s.} \right)^2 \]
\[ \frac{15369.3605}{d^2} + \left( -\frac{2636.9655}{d^2} \right)^2 - 2 \times \frac{15369.3605}{d^2} \times \frac{-2636.9655}{d^2} \leq \left( \frac{300}{2.0} \right)^2 \]
\[ \frac{18006.326}{d^2} \leq 150; \]
\[ d \geq \sqrt{\frac{18006.326}{150}}; \]
\[ d \geq 10.9564 \text{ mm}; \]

Q. A wall bracket, as shown in following figure, is subjected to a pull of \( P = 5 \text{ kN} \), at 60° to the vertical. The cross-section of bracket is rectangular having \( b = 3t \). Determine the dimensions \( b \) and \( t \) if the stress in the material of the bracket is limited to 28 MPa.
**Solution:**

Given: \( P = 6000 \); \( \sigma = 60 \text{ MPa} = 60 \text{ N/mm}^2 \)

Let \( t \) = Thickness of the section in mm, and 
\( b \) = Depth or width of the section = 3 \( t \)

Area of cross-section,  
\[ A = b \times t = 3t \times t = 3t^2 \text{ mm}^2 \]

and section modulus,  
\[ Z = \frac{tb^3}{6} = \frac{3t^3}{2} \]

Horizontal component of the load,  
\( PH = 5000 \sin 60^\circ \)  
\[ = 5000 \times 0.866 = 4330.13 \text{ N} \]

Bending moment due to horizontal component of the load,  
\( MH = PH \times 60 = 4330.13 \times 60 = 259807.62 \text{ N-mm} \)

Maximum bending stress on the upper surface due to horizontal component,  
\[ \sigma_{bh} = \frac{MH}{z} = \frac{259807.62 \times 2}{3t^2} = \frac{173205.81}{t^2} \text{ N/mm}^2 \]

Vertical component of the load,  
\( PV = 5000 \cos 60^\circ = 6000 \times 0.5 = 2500 \text{ N} \)

Direct Shear;  
\[ \tau = \frac{PV}{A} = \frac{2500}{3t^2} = \frac{833.33}{t^2} \text{ N/mm}^2 \]

Bending moment due to vertical component of the load,  
\( MV = PV \times 60 = 2500 \times 120 = 300000 \text{ N-mm} \)

Maximum bending stress on the upper surface due to horizontal component,  
\[ \sigma_{br} = \frac{MV}{z} = \frac{300000 \times 2}{3t^2} = \frac{200000}{t^2} \text{ N/mm}^2 \]

Direct tensile stress due to horizontal component  
\[ \sigma_d = \frac{PH}{A} = \frac{4330.13}{3t^2} = \frac{1443.38}{t^2} \text{ N/mm}^2 \]

Net normal stress  
\[ \sigma = \frac{173205.81}{t^2} + \frac{200000}{t^2} + \frac{1443.38}{t^2} = \frac{374649.1867}{t^2} \text{ N/mm}^2 \]

Now applying the maximum shear stress theory
\[ \frac{1}{2} \sqrt{\left( \sigma^2 + 4\tau^2 \right)} \leq 28 \]

*putting the values and solving the above equation for "t"*

\[ t = 25\text{mm} \text{ and } b = 3t = 75\text{ mm} \]
In this chapter we will be discussing on design aspects related to fatigue failure, an important mode of failure in engineering components. Fatigue failure results mainly due to variable loading or more precisely due to cyclic variations in the applied loading or induced stresses. So starting from the basic concepts of variable (non-static) loading, we will be discussing in detail how it leads to fatigue failure in components, what factors influence them, how to account them, and finally how to design parts or components to resist failure by fatigue.

**FATIGUE:**

Fatigue is a phenomenon associated with variable loading or more precisely to cyclic stressing or straining of a material. Just as we human beings get fatigue when a specific task is repeatedly performed, in a similar manner metallic components subjected to variable loading get fatigue, which leads to their premature failure under specific conditions.

**WHAT IS FATIGUE LOADING?**

Fatigue loading is primarily the type of loading which causes cyclic variations in the applied stress or strain on a component. Thus any variable loading is basically a fatigue loading.

In reality most mechanical components experience variable loading due to:

- Change in the magnitude of applied load Example: punching or shearing operations
- Change in direction of load application Example: a connecting rod
- Change in point of load application Example: a rotating shaft

There are different types of fatigue/variable loading. The worst case of fatigue loading is the case known as *fully-reversible load*. One cycle of this type of loading occurs when a tensile stress of some value is applied to an unloaded part and then released, then a compressive stress of the same value is applied and released.

**Variable Loading**

Variable loading results when the applied load or the induced stress on a component is not constant but changes with time i.e load or stress varies with time in some pattern. Most mechanical systems and devices consist of moving or rotating components. When they are subjected to external loadings, the induced stresses are not constant even if the magnitude of the applied load remains invariant.

In reality most mechanical components experience variable loading due to

- Change in the magnitude of applied load Example: punching or shearing operations
- Change in direction of load application Example: a connecting rod
There are different types of fatigue/variable loading. The worst case of fatigue loading is the case known as \textit{fully-reversible load}. One \textit{cycle} of this type of loading occurs when a tensile stress of some value is applied to an unloaded part and then released, then a compressive stress of the same value is applied and released.

A rotating shaft with a bending load applied to it is a good example of fully reversible load. In order to visualize the fully-reversing nature of the load, picture the shaft in a fixed position (not rotating) but subjected to an applied bending load (as shown here). The outermost fibers on the shaft surface lying on the convex side of the deflection (upper surface in the picture) will be loaded in tension (upper green arrows), and the fibers on the opposite side will be loaded in compression (lower green arrows). Now, rotate the shaft 180° in its bearings, with the loads remaining the same. The shaft stress level is the same, but now the fibers which were loaded in compression before you rotated it are now loaded in tension, and vice-versa. Thus if the shaft is rotated let us say at 900 revolutions per minute then the shaft is cyclically stressed 900 times a minute.

To illustrate how damaging such type load is, take a paper clip, bend it out straight, then pick a spot in the middle, and bend the clip 90° back and forth at that spot (from straight to "L" shaped and back). When you bend it the other way, you reverse the stresses (fully reversing fatigue). You can notice that the clip will break in a few to about a maximum of 10 cycles.

When you are bending it you are plastically-deforming the metal, you are, by definition, exceeding its yield stress. When you bend it in one direction, you are applying a high tensile stress to the fibers on one side of the OD, and a high compressive stress on the fibers on the opposite side. In the next cycle the phenomena is repeated, the tensile stress fibers are now compressed and vice versa, thus the material is cyclically strained which ultimately results in their premature failure.

\textbf{Fatigue Failure}

Often machine members subjected to such repeated or cyclic stressing are found to have failed even when the actual maximum stresses were below the ultimate strength of the material, and quite frequently at stress values even below the yield strength. The most distinguishing characteristics is that the failure had occurred only after the stresses have been repeated a very large number of times. Hence the failure is called fatigue failure.

\textbf{Fatigue Failure- Mechanism}

A fatigue failure begins with a small crack; the initial crack may be so minute and can not be detected. The crack usually develops at a point of localized stress concentration like discontinuity in the material, such as a change in cross section, a keyway or a hole. Once a crack is initiated, the stress concentration effect become greater and the crack propagates. Consequently the stressed area decreases in size, the stress increase in magnitude and the
crack propagates more rapidly. Until finally, the remaining area is unable to sustain the load and the component fails suddenly. Thus fatigue loading results in sudden, unwarned failure.

**Fatigue Failure Stages**

Thus three stages are involved in fatigue failure namely
- Crack initiation
- Crack propagation
- Fracture

The macro mechanism of fatigue failure is briefly presented now.

**Crack initiation**

- Areas of localized stress concentrations such as fillets, notches, key ways, bolt holes and even scratches or tool marks are potential zones for crack initiation.
- Crack also generally originate from a geometrical discontinuity or metallurgical stress raiser like sites of inclusions
- As a result of the local stress concentrations at these locations, the induced stress goes above the yield strength (in normal ductile materials) and cyclic plastic straining results due to cyclic variations in the stresses.

On a macro scale the average value of the induced stress might still be below the yield strength of the material.
- During plastic straining slip occurs and (dislocation movements) results in gliding of planes one over the other. During the cyclic stressing, slip saturation results which make further plastic deformation difficult.
- As a consequence, intrusion and extrusion occurs creating a notch like discontinuity in the material.

**Crack propagation**

- This further increases the stress levels and the process continues, propagating the cracks across the grains or along the grain boundaries, slowly increasing the crack size.
- As the size of the crack increases the cross sectional area resisting the applied stress decreases and reaches a threshold level at which it is insufficient to resist the applied stress.

**Final fracture**

- As the area becomes too insufficient to resist the induced stresses any further a sudden fracture results in the component.

**How is the fatigue strength of a metal determined?**

The fatigue behavior of a specific material, heat-treated to a specific strength level, is determined by a series of laboratory tests on a large number of apparently identical samples of that specific material. These laboratory samples are optimized for fatigue life. These laboratory samples are now standardized in geometry and configuration such that no extraneous factors other than the applied stress influence the fatigue life. They are machined with shape characteristics which maximize the fatigue life of a metal, and are highly polished to provide the surface characteristics which enable the best fatigue life. A single test consists of applying a known, constant bending stress to a round sample of the material, and rotating the sample
around the bending stress axis until it fails. As the sample rotates, the stress applied to any
fiber on the outside surface of the sample varies from maximum-tensile to zero to
maximum-compressive and back. The test mechanism counts the number of rotations
(cycles) until the specimen fails. A large number of tests is run at each stress level of
interest, and the results are statistically massaged to determine the expected number of
cycles to failure at that stress level. The most widely used fatigue-testing device is the R.R
Moore high-speed rotating beam machine. This machine subjects the specimen to pure
bending (no transverse shear).

The S-N Diagram
Tests on several specimens are conducted under identical conditions with varying levels of
stress amplitude. The cyclic stress level of the first set of tests is some large percentage of
the Ultimate Tensile Stress (UTS), which produces failure in a relatively small number of
cycles. Subsequent tests are run at lower cyclic stress values until a level is found at which
the samples will survive 10 million cycles without failure.

The results are plotted as an S-N diagram (see the figure) usually on semi-log or on log-log
paper, depicting the life in number of cycles tested as a function of the stress amplitude. A
typical plot is shown in the figure below for two class of materials.
**Endurance or Fatigue Limit**

In the case of the steels, a knee (flattening or saturation) occurs in the graph, and beyond this knee failure will not occur, no matter how large the numbers of cycles are. The strength (stress amplitude value) corresponding to the knee is called the endurance limit (Se) or the fatigue limit. However the graph never does become horizontal for non-ferrous metals and alloys, hence these materials do not have an endurance limit.

**Endurance or Fatigue limit - definition**

Endurance or fatigue limit can be defined as the magnitude of stress amplitude value at or below which no fatigue failure will occur, no matter how large the number of stress reversals are, in other words leading to an infinite life to the component or part being stressed. For most ferrous materials Endurance limit (Se) is set as the cyclic stress level that the material can sustain for 10 million cycles.

In general, steel alloys which are subjected to a cyclic stress level below the EL (properly adjusted for the specifics of the application) will not fail in fatigue. That property is commonly known as "infinite life". Most steel alloys exhibit the infinite life property, but it is interesting to note that most aluminum alloys as well as steels which have been casehardened by carburizing, do not exhibit an infinite-life cyclic stress level (Endurance Limit).

**S-N Diagram**

To find the fatigue strength and the endurance limit of a material, quite a large number of tests are necessary because of the statistical nature of fatigue. For the first test in the high speed rotating beam test, the bending moment is kept slightly less than the ultimate tensile strength of the material. Then the stress is slightly reduced and a second test is carried out. This process is continued and the results are plotted as fatigue strength “$S_f$” versus stress cycles “$N$” popularly known as S-N Diagram as shown above, on log –log paper. For the ferrous materials, like steels the graph becomes horizontal at $10^6$ cycles indicating the fatigue failure will not occur beyond this whatever great may be the number of the cycles. The corresponding fatigue strength is known as endurance limit of the material.
Low Cycle Fatigue
The body of knowledge available on fatigue failure from N=1 to N=1000 cycles is generally classified as low-cycle fatigue.

High Cycle Fatigue
High-cycle fatigue, then, is concerned with failure corresponding to stress cycles greater than $10^3$ cycles. (Note that a stress cycle (N=1) constitutes a single application and removal of a load and then another application and removal of load in the opposite direction. Thus N=½ means that the load is applied once and then removed, which is the case with the simple tensile test.)

Finite and Infinite Life
We also distinguish a finite-life and an infinite-life region. Finite life region covers life in terms of number of stress reversals up to the knee point (in case of steels) beyond which is the infinite-life region. The boundary between these regions cannot be clearly defined except for specific materials; but it lies somewhere between 106 and 107 cycles, for materials exhibiting fatigue limit.

Factors Considered While Designing Against Fatigue: Any material against fatigue loading is designed in the two ways i.e design for finite life and design for infinite life. The design for finite life is based on the fatigue strength whereas the design for infinite life is based on the endurance limit. Hence in both the cases the following modifying factors should be considered while designing against the fatigue:

- Size factor;
- Surface finish factor;
- Temperature Factor;
- Reliability factor
- Modifying factor for stress concentration
- Environmental factor
- Application factor etc.

In addition to above factor the type of loading that is repeated, fluctuating or completely reversed load.

Size Factor: A little consideration will show that if the size of the standard specimen is increased, then the endurance limit of the material will decrease. This is due to the fact that a longer specimen will have more defects than a smaller one.

Let $K_{sz} =$ Size factor.

1. The value of size factor is taken as unity for the standard specimen having nominal diameter of 7.657 mm.
2. When the nominal diameter of the specimen is more than 7.657 mm but less than 50 mm, the value of size factor may be taken as 0.85.
3. When the nominal diameter of the specimen is more than 50 mm, then the value of size factor may be taken as 0.75.
**Surface Finish Factor:** When a machine member is subjected to variable loads, the endurance limit of the material for that member depends upon the surface conditions, as shown in the following figure, the values of surface finish factor for the various surface conditions and ultimate tensile strength. When the surface finish factor is known, then the endurance limit for the material of the machine member may be obtained by multiplying the endurance limit and the surface finish factor. We see that for a mirror polished material, the surface finish factor is unity. In other words, the endurance limit for and for non ferrous materials it can be taken as unity.

Let $K_{sur} = $ Surface finish factor.

![Surface Finish Factor Diagram](image)

**Figure 2.4**

**Stress Concentration Effect**

Localization of high magnitude stresses in the vicinity of any irregularity or discontinuity or abrupt change in the cross section of the mechanical component is known as stress concentration effect.

**Fatigue Stress Concentration**

The existence of irregularities or discontinuities, such as holes, grooves, or notches, in a part increase the magnitude of stresses significantly in the immediate vicinity of the discontinuity. Fatigue failure mostly originates from such places. Hence its effect must be accounted and normally a fatigue stress-concentration factor $K_I$ is applied when designing against fatigue, even if the materials behaviour is ductile.
Fatigue Stress Concentration Factor

Recall that a stress concentration factor need not be used with ductile materials when they are subjected to only static loads, because (local) yielding will relieve the stress concentration. However under fatigue loading, the response of material may not be adequate to nullify the effect and hence has to be accounted. The factor \( K_f \) commonly called a fatigue stress concentration factor is used for this. Normally, this factor is used to indicate the increase in the stress; hence this factor is defined in the following manner. Fatigue stress concentration factor can be defined as

\[
K_f = \frac{\text{Endurance limit without stress concentration}}{\text{Endurance limit with stress concentration}}
\]

**Notch Sensitivity:** It may be defined as the degree to which the theoretical effect of stress concentration is actually reached.

**Notch Sensitivity Factor “q”:** Notch sensitivity factor is defined as the ratio of increase in the actual stress to the increase in the nominal stress near the discontinuity in the specimen.

\[
q = \frac{K_f - 1}{K_t - 1}
\]

Where, \( K_f \) and \( K_t \) are the fatigue stress concentration factor and theoretical stress concentration factor.
Methods of reducing the stress concentration are as follows:

The presence of stress concentration cannot be totally eliminated but it may be reduced to some extent.

To reduce the stress concentration, the stress lines should maintain their spacing as far as possible.

The change in the cross section should be gradual to the possible extent. The following figures shows the use of notches, drilled holes or removing the extra material, which may be adopted in the design in order to reduce the stress concentration effect. The stress concentration effect can also be reduced by improving the surface finish.

**Figure 2.6**

Relation Between Endurance Limit and Ultimate Tensile Strength

It has been found experimentally that endurance limit ($\sigma_e$) of a material subjected to fatigue loading is a function of ultimate tensile strength ($\sigma_u$). The following figure shows the endurance limit of steel corresponding to ultimate tensile strength for different surface conditions. Following are some empirical relations commonly used in practice:

For steel, $\sigma_e = 0.5 \sigma_u$;
For cast steel, $\sigma_e = 0.4 \sigma_u$;
For cast iron, $\sigma_e = 0.35 \sigma_u$;
For non-ferrous metals and alloys, $\sigma_e = 0.3 \sigma_u$.

**Figure 2.7**
Goodman Method for Combination of Stresses:
A straight line connecting the endurance limit ($\sigma_e$) and the ultimate strength ($\sigma_u$), as shown by line $AB$ in figure given below follows the suggestion of Goodman. A Goodman line is used when the design is based on ultimate strength and may be used for ductile or brittle materials.

![Diagram](image)

**Figure 2.8**

Now from similar triangles COD and PQD,

$$\frac{PQ}{CO} = \frac{QD}{OD} = \frac{OD - OQ}{OD} = 1 - \frac{OQ}{OD}$$

...($\because$ $OD = OD - OQ$)

$$\therefore \frac{\sigma_v}{\sigma_e / F.S.} = 1 - \frac{\sigma_m}{\sigma_u / F.S.}$$

$$\sigma_v = \frac{\sigma_e}{F.S.} \left[ 1 - \frac{\sigma_m}{\sigma_u / F.S.} \right] = \sigma_e \left[ 1 - \frac{\sigma_m}{\sigma_u} \right]$$

Or

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v}{\sigma_e}$$

...($i$)

This expression does not include the effect of stress concentration. It may be noted that for ductile materials, the stress concentration may be ignored under steady loads. Since many machine and structural parts that are subjected to fatigue loads contain regions of high stress concentration, therefore equation ($i$) must be altered to include this effect. In such cases, the fatigue stress concentration factor ($K_f$) is used to multiply the variable stress ($\sigma_v$). The equation ($i$) may now be written as


\[
\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e}
\]

...(ii)

where

- \(F.S.\) = Factor of safety,
- \(\sigma_m\) = Mean stress,
- \(\sigma_u\) = Ultimate stress,
- \(\sigma_v\) = Variable stress,
- \(\sigma_e\) = Endurance limit for reversed loading, and
- \(K_f\) = Fatigue stress concentration factor.

Considering the load factor, surface finish factor and size factor, the equation (ii) may be written as

\[
\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{b} \times K_{sur} \times K_{sz}}
\]

...(iii)

\[
= \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}}
\]

...(\because \sigma_{eb} = \sigma_e \times K_{b} and K_{b} = 1)

where

- \(K_b\) = Load factor for reversed bending load,
- \(K_{sur}\) = Surface finish factor, and
- \(K_{sz}\) = Size factor.

**Soderberg Method for Combination of Stresses**

A straight line connecting the endurance limit (\(\sigma_e\)) and the yield strength (\(\sigma_y\)), as shown by the line \(AB\) in following figure, follows the suggestion of Soderberg line. This line is used when the design is based on yield strength. The line \(AB\) connecting \(\sigma_e\) and \(\sigma_y\), as shown in following figure, is called **Soderberg’s failure stress line**. If a suitable factor of safety \((F.S.)\) is applied to the endurance limit and yield strength, a safe stress line \(CD\) may be drawn parallel to the line \(AB\). Let us consider a design point \(P\) on the line \(CD\). Now from similar triangles \(COD\) and \(PQD\),

**Figure 2.9**
Modified Goodman Diagram:
In the design of components subjected to fluctuating stresses, the Goodman diagram is slightly modified to account for the yielding failure of the components, especially, at higher values of the mean stresses. The diagram known as modified Goodman diagram and is most widely used in the design of the components subjected to fluctuating stresses. There are two modified Goodman diagrams for the axial, normal or bending stresses and shear or torsion shear stresses separately as shown below. In the following diagrams the safe zones are ABCOA.

![Modified Goodman Diagrams](image1)

**Figure 2.10**

**DESIGN APPROACH FOR FATIGUE LOADINGS**

**Design for Infinite Life**
It has been noted that if a plot is made of the applied stress amplitude verses the number of reversals to failure to (S-N curve) the following behaviour is typically observed.

![S-N Curve](image2)

**Figure 2.11**
**Completely Reversible Loading**

If the stress is below the (the endurance limit or fatigue limit), the component has effectively infinite life. for the most steel and copper alloys. If the material does not have a well defined $\sigma_e$. Then, endurance limit is arbitrarily defined as Stress(0.35- 0.50) that gives For a known load (Moment ) the section area/(modulus) will be designed such that the resulting amplitude stress will be well below the endurance limit.

Design approach can be better learnt by solving a problem.

**Q.** A machine component is subjected to bending stress which fluctuates between 300 N/mm$^2$ tensile and 150 N/mm$^2$ compressive in cyclic manner. Using the Goodman and Soderberg criterion, calculate the minimum required ultimate tensile strength of the material. Take the factor of safety 1.5 and the endurance limit in reversed bending as 50% of ultimate tensile strength.

**Solution:**

Assuming the yield strength $S_{yt} = 0.55 \times$ ultimate strength $S_{ut}$

Mean stress $\sigma_m = \frac{\sigma_{\text{max}} + \sigma_{\text{min}}}{2} = \frac{300 + (-150)}{2} = 75\text{MPa}$;

Amplitude stress $\sigma_a = \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2} = \frac{300 - (-150)}{2} = 225\text{MPa}$;

As per Goodman Relation:

$$\frac{1}{\text{f.o.s}} = \frac{\sigma_m}{S_{\text{ut}}} + \frac{\sigma_a}{S_e}$$

As given $S_e = 0.5S_{\text{ut}}$

$$\frac{1}{1.5} = \frac{75}{S_{\text{ut}}} + \frac{225}{0.5S_{\text{ut}}} \Rightarrow \frac{1}{1.5} = \frac{525}{S_{\text{ut}}} \Rightarrow S_{\text{ut}} = 787.5\text{MPa};$$

As per Soderberg Relation:

$$\frac{1}{\text{f.o.s}} = \frac{\sigma_m}{S_{\text{yt}}} + \frac{\sigma_a}{S_e}$$

As given $S_{\text{yt}} = 0.55S_{\text{ut}}$

$$\frac{1}{1.5} = \frac{75}{0.55S_{\text{ut}}} + \frac{225}{0.5S_{\text{ut}}} \Rightarrow \frac{1}{1.5} = \frac{586.36}{S_{\text{ut}}} \Rightarrow S_{\text{ut}} = 879.545\text{MPa};$$

**Q.** A circular bar is subjected to a completely reversed axial load of 150 kN. Determine the size of the bar for infinite life, if it is made of plain carbon steel having ultimate tensile strength of 800 N/mm$^2$ and yield point in tension of 600 N/mm$^2$. Assuming the surface finish factor as 0.80, size factor 0.85, reliability as 90%, and modifying factor for the stress concentration as 0.9.
Solution:

**GIVEN:**

Maximum Axial Load \( P_{\text{max}} = +150 \text{kN} \);
Minimum Axial Load \( P_{\text{min}} = -150 \text{kN} \);
Ultimate tensile strength of the material of the bar \( S_{\text{ut}} = 800 \ \text{N/mm}^2 \);
Yield point in tension of the material of the bar \( S_{\text{yt}} = 600 \ \text{N/mm}^2 \);
Surface finish factor \( k_d = 0.80 \); Size factor \( k_b = 0.85 \); Reliability factor \( k_c = 0.90 \); Modifying the stress concentration factor \( k_e = 0.90 \).

**ASSUMING:**

The temperature factor \( k_d = 1.0 \); & miscellaneous factor \( k_g = 1.0 \);
Factor of safety = 1.0;

**Endurance Limit of the material:**

\[ S_e = 0.5 \times S_{\text{ut}} = 0.5 \times 800 = 400 \text{MPa} \]

**Modified Endurance Limit of the Material of the Bar:**

\[ S_e = k_a \times k_b \times k_c \times k_d \times k_e \times S_e = 0.80 \times 0.85 \times 0.90 \times 1.0 \times 0.9 \times 1.0 \times 400 = 220.32 \text{ MPa} \]

**Amplitude and Mean Normal Stresses:**

Amplitude Load \( P_a = \frac{P_{\text{max}} - P_{\text{min}}}{2} = \frac{150 - (-150)}{2} = 150 \text{kN} \);
Mean Load \( P_m = \frac{P_{\text{max}} + P_{\text{min}}}{2} = \frac{150 + (-150)}{2} = 0 \text{kN} \);
Amplitude Stress \( \sigma_a = \frac{4 \times P_a}{\pi \times d^2} = \frac{4 \times 150}{\pi \times d^2} = \frac{190.9859}{d^2} \text{ kN} / \text{mm}^2 = \frac{190985.9}{d^2} \text{ N/mm}^2 \);
Mean Stress \( \sigma_m = 0 \text{ N/mm}^2 \);

**Using Modified Goodman Diagram:**
The Load Line becomes the amplitude axis.
Hence the design equation may be written for infinite life as:

\[ S_e \geq \sigma_a \Rightarrow 220.32 = \frac{190985.9}{d^2} \]

\[ d \geq \sqrt{\frac{190985.9}{220.32}} \Rightarrow d \geq 29.44 \]

\( d = 30 \text{mm} \).

(b) A cantilever beam made of cold drawn steel 20C8 \( (S_{\text{ut}} = 540 \text{ N/mm}^2) \) is subjected to a completely reversed load of 1000 N as shown in below figure. The corrected endurance limit for the material of the beam may be taken as 123.8 N/mm². Determine the diameter “d” of the beam for a life of 10000 cycles.
Solution:

**GIVEN:**
- Maximum Axial Load $P_{\text{max}} = +1000$ N;  
- Minimum Axial Load $P_{\text{min}} = -1000$ N;  
- Ultimate tensile strength of the material of the bar $S_{\text{ut}} = 540$ N/mm$^2$;  
- Corrected Endurance Limit $S_e = 123.8$ N/mm$^2$.

**USING THE S-N DIAGRAM:**

The values of various points:
- $0.9S_{\text{ut}} = 0.9 \times 540 = 486$ N/mm$^2$;  
- $\log_{10}(0.9S_{\text{ut}}) = \log_{10}(486) = 2.6866$;  
- $\log_{10}(S_e) = \log_{10}(123.8) = 2.0927$;  
- $\log_{10}(N) = \log_{10}(10000) = 4.0$.

**From the S-N Diagram:**

$$\frac{\overline{AE}}{\overline{DB}} = \frac{\overline{AD} \times \overline{EF}}{(6 - 3)} = \frac{(2.6866 - 2.0927) \times (4 - 3)}{3} = 0.198$$

Therefore:
- $\log_{10} S_f = 2.6866 - \overline{AE} = 2.6866 - 0.198 = 2.4886$;  
- $S_f = 308.03$ N/mm$^2$;

And
- $S_f = \sigma_b = \frac{32M_b}{\pi d^3} \Rightarrow d^3 = \frac{32 \times M_b}{\pi \times S_f} = \frac{32 \times (1000 \times 150)}{\pi \times 308.3} = \frac{479.79}{17.05}$.

Therefore, $d = 17.05$ mm.

Q. A flat bar as shown in the figure 1 is subjected to an axial load $F$ equal to 500 N. Assuming that the stress in the bar is limited to 200 MPa, determine the thickness of the bar. All dimensions are in mm.
**Solution:** The stress concentration factor at circular hole $k_t = 2.35$

The stress concentration factor at circular hole $k_t = 1.78$

Hence the critical section is at the section of circular hole. The stress magnitude induced at this location

$$\sigma = \frac{F}{t \times (100 - 30)} = \frac{500}{t \times 70} = \frac{7.143}{t} N/mm^2;$$

For successful design

$$\frac{7.143}{t} \leq 200;$$
$$t \geq 0.036 mm;$$

However the minimum cross section area “A” = 62 mm$^2$;

$$\sigma = \frac{F}{62t} = \frac{500}{t \times 62} = \frac{8.06452}{t} N/mm^2;$$

For successful design

$$\frac{8.06452}{t} \leq 200;$$
$$t \geq 0.0403 mm;$$

Hence the thickness of the plate = 0.0403 mm.

**Q.** A forged steel bar 50 mm in diameter is subjected to a reversed bending stress of 300 MPa. The bar is made of 40C8. Calculate the life of the bar for a reliability of 90%

**Given:** The material 40C8,
*The diameter of the shaft = d=50mm;
Reversed bending stress = 300MPa.
Reliability = 90%*

**Assuming:** $\sigma_{ut}= 600MPa$ ; $\sigma_{yt}=380 MPa$ Fatigue stress concentration factor=1.612 ;

Assuming the size factor $Ksz = 0.85$ ;
Surface finish factor $Ksur = 0.89$
Reliability factor $Kre=0.892$

**Endurance Limit:**

$$S'_e = 0.5 \sigma_{ut} \quad \therefore \sigma_{ut} < 1400 MPa;$$
$$S'_e = 0.5 \times 600 = 300 MPa;$$

**Modified Endurance Limit:**
Using the S-N diagram

\[ \log_{10}(S_{ut}) = \log_{10}(600) = 2.778; \]
\[ 0.9 \times \log_{10}(S_{ut}) = 2.500 \]
\[ \log_{10}(S_c) = \log_{10}(125.583) = 2.099; \]
\[ \log_{10}(\sigma_a) = \log_{10}(300) = 2.477; \]
\[ \frac{2.50 - 2.099}{6 - 3} = \frac{2.477 - 2.099}{6 - \log_{10} N} \]
\[ 6 - \log_{10} N = 2.82793 \]
\[ \log_{10} N = 6 - 2.82793 = 3.17207 \]
\[ N = 1486.1752 \text{ reversals} \]

Q. A shaft subjected to bending moment varying from -200 N m to +500 N m and a varying torque from 50 N m to 175 N m. If material of the shaft is 30C8, stress concentration factor is 1.85, notch sensitivity is 0.95 reliability 99.9% and factor of safety is 1.5, find the diameter of the shaft.

**Solution:** Mean or average bending moment,

\[ M_m = \frac{M_{\text{max}} + M_{\text{min}}}{2} = \frac{500 + (-200)}{2} = 150 N \cdot m; \]

Amplitude or variable bending moment

\[ M_a = \frac{M_{\text{max}} - M_{\text{min}}}{2} = \frac{500 - (-200)}{2} = 350 N \cdot m \]

Mean or average torque,

\[ T_m = \frac{T_{\text{max}} + T_{\text{min}}}{2} = \frac{175 + (50)}{2} = 112.5 N \cdot m; \]

Amplitude or variable torque

\[ T_a = \frac{T_{\text{max}} - T_{\text{min}}}{2} = \frac{175 - (50)}{2} = 62.5 N \cdot m \]

**Equivalent mean and amplitude bending moments**

\[ M_{cm} = \frac{1}{2} \left[ M_m + \sqrt{M_m^2 + M_a^2} \right] = \frac{1}{2} \left[ 150 + \sqrt{150^2 + 350^2} \right] = 168.75 \text{N} \cdot \text{m} = 168750 \text{N} \cdot \text{mm}; \]
\[ M_{am} = \frac{1}{2} \left[ M_a + \sqrt{M_a^2 + T_a^2} \right] = \frac{1}{2} \left[ 350 + \sqrt{350^2 + 62.5^2} \right] = 352.77 \text{N} \cdot \text{m} = 352770 \text{N} \cdot \text{mm}; \]
Mean or average bending stress,
\[ \sigma_m = \frac{32M_{em}}{\pi \times d^3} = \frac{32 \times 168750}{\pi \times d^3} = \frac{171874}{d^3}; \]

Amplitude or variable bending moment
\[ \sigma_a = \frac{32M_{am}}{\pi \times d^3} = \frac{32 \times 352.77}{\pi \times d^3} = \frac{359328.6}{d^3}; \]

Material properties: \( \sigma_{ut} = 490 \text{MPa} \); \( \sigma_{yt} = 270 \text{MPa} \) (assumed)

Given Notch sensitivity \( q = 0.95 \);

Assuming the size factor \( K_{sz} = 0.85 \);

Surface finish factor \( K_{sur} = 0.89 \)

Reliability factor \( K_{re} = 0.75 \) corresponding to 99.9% reliability (Assumed)

The fatigue stress concentration factor
\[ K_f = 1 + q(K_r - 1) = 1 + 0.95(1.85 - 1) = 1.8075 \]

**Endurance Limit:**
\[ S'_e = 0.5\sigma_{ut} \quad : \quad \sigma_{ut} < 1400 \text{MPa}; \]
\[ S'_e = 0.5 \times 490 = 245 \text{MPa}; \]

**Modified Endurance Limit:**
\[ S_e = K_{sz} \times K_{sur} \times K_{re} \times \frac{1}{K_f} \times S'_e = 0.85 \times 0.89 \times 0.75 \times \frac{1}{1.8075} \times 245 = 76.91 \text{MPa} \]

We know that according to Soderberg formula;
\[
\frac{1}{f.o.s.} = \frac{\sigma_m}{\sigma_{yt}} + \frac{\sigma_a}{S_e} ;
\]
\[
\frac{1}{1.5} = \frac{171874}{270} + \frac{359328.6}{76.91};
\]
\[
d^3 = \frac{171874}{270} + \frac{359328.6}{76.91};
\]
\[
d^3 = 1.5 \times [100519 + 6083178] = 79630.291
\]
\[
d = 43.022 \text{ mm} \approx 45 \text{ mm};
\]

We know that according to Goodman's formula
\[
\frac{1}{f.o.s.} = \frac{\sigma_m}{\sigma_{ut}} + \frac{\sigma_{a.}}{S_v};
\]
\[
1 = \frac{d^3}{490} + \frac{d^3}{76.91};
\]
\[
d^3 = \frac{1718874}{490} + \frac{3593286}{76.91};
\]
\[
d^3 = 75342.85
\]
\[
d = 42.235\text{mm} \approx 45\text{ mm};
\]

Hence the shaft diameter is 45 mm. Ans.
REVITED JOINT

What is joint?

The means to keep the two things together is known as joint.

Components of any Joint:

There are two basic components of any joint:
1. The components to be joined
2. Fastener or joining elements

Types of Joints:

1. Permanent Joints;
2. Semi-Permanent Joints;
3. Temporary Joints.

Permanent Joint:

If the joined or fastened element joint can not open or separated without causing any damage them, then joint is known as permanent joint. Like Welded Joints

Semi-Permanent Joint:

If the joined or fastened element joint can open or separated with a marginal damage to any one of the joined parts, then joint is known as semi-permanent joint. Like Riveted Joints

Temporary Joint:

If the joined or fastened element joint can open or separated without causing any damage them, then joint is known as temporary joints. Like Bolted Joints

Components of Riveted Joints: There are two basic components of riveted joints:
1. Rivets and;
2. Two or more plates.

Rivet: Rivet is a cylindrical bar having a head and tail portion as its integral part.

Material Used for Rivets: The rivet head are made by the following two methods:
- Cold Heading and;
- Hot forging;

Therefore the material of the rivets must possess the sufficient DUCTILITY and TOUGHNESS.

The popular materials for the rivets are:
Steel, Brass, Aluminium & Copper.

as per the requirement of the application for fluid tight joints the steel rivets are used

**AS per Indian standard IS: 2998-1982:**

The material of rivet must have the following properties:

- Minimum tensile strength = 40 N/mm²; And
- Elongation capacity not less than 26%.

**TYPES OF RIVET HEADS**

![Figure 2.12](image)

**Methods of Riveting**
If the shank diameter of the rivet is more than 12 mm than machine riveting is preferred for cold riveting and if the shank diameter of the rivet is more than 18 mm Hot machine riveting is preferred.

**Advantages of Riveted Joints**

1. Production rates are high.
2. Initial and maintenance costs for equipment are low.
3. Either metallic or non-metallic materials can be joined.
4. Dissimilar metals and assemblies having a number of parts with non-uniform thickness can readily be fastened.
5. Unskilled labor can be used to operate equipment.
6. The rivets can be made of a variety of material ranging from monel or inconel to lead or zinc.
7. The rivets can be used not only as a fastener but as a pivot or other functional component.

**Limitations of Riveted Joints:**

1. In riveted joints the parts once fastened can not be easily disassembled.
2. The riveted joints are not as fluid tight as welded joints.
3. The riveted joints weaken the parts to be connected.
4. The complicated components can not be riveted.
5. The riveted joints can not be produced as fast as welded joints.
6. As compared to welded joints the riveted joints lead to bulky construction.
7. The protruding rivet heads are inconvenient and undesirable in many applications.

**TYPES OF RIVETED JOINTS:**

**Lap Joint:**

In a lap joint, the plates to be connected overlap each other and the rivet pass through drilled coaxial holes in the overlapped plates.

**Butt Joint:**

In butt riveting or joint, the plates are kept in alignment and a butt strap or a cover plate (either single or double) is placed over the joint and rivets are inserted through holes in plates aligned over one another.
Figure 2.13

Figure 2.14
Caulking and Fullering:

In order to make the joints leak proof or fluid tight in pressure vessels like steam boilers, air receivers and tanks etc., a process known as caulking is employed. In this process, a narrow blunt tool called caulking tool, about 5 mm thick and 38 mm in breadth, is used. The edge of the tool is ground to an angle of 80°. The tool is moved after each blow along the edge of the plate, which is planed to a bevel of 75° to 80° to facilitate the forcing down of edge. It is seen that the tool burrs down the plate at A in Fig. (a) forming a metal to metal joint. In actual practice, both the edges at A and B are caulked. The head of the rivets as shown at C are also turned down with a caulking tool to make a joint steam tight. A great care is taken to prevent injury to the plate below the tool.
A more satisfactory way of making the joints staunch is known as **fullering** which has largely superseded caulking. In this case, a fullering tool with a thickness at the end equal to that of the plate is used in such a way that the greatest pressure due to the blows occur near the joint, giving a clean finish, with less risk of damaging the plate. A fullering process is shown in Fig.(b).

![Figure 2.14](image)

**Modes of Failure of Riveted Joint**

The failures of riveted joint can be either:

1. Failure of Plate
2. Failure of Rivets

**Strength of the Riveted Joints:** The strength of a riveted joint is defined as the maximum force which the joint can take without failure per pitch length.

\[
\text{Strength of riveted joint} \leq \text{least among } P_v, P_r, \text{ and } P_c
\]
Efficiency of the Riveted Joint:

The efficiency of a riveted joint is the ratio of the strength of the joint to the strength of an un-riveted or solid plate.

\[ \eta = \frac{\text{Strength of riveted joint}}{\text{Strength of an un-riveted or solid plate}} \]

\[ \text{Efficiency of riveted joint} = \frac{\text{Minimum of } P_i, P_s, \text{ and } P_c}{\text{Minimum of } P_i, P_s, \text{ and } P_c} \]

Q. A bracket supported by means of four rivets of the same size as shown in below figure. Determine the diameter of the rivets if the maximum permissible shear stress for the material of the rivet is as 150 N/mm².

Solution:

Given:

\[ W = 20000 \text{ N}; \ n = 4; \ \tau_{\text{all}} = 150 \text{ N/mm}^2; \ e = 80 \text{ mm}; \]

1. C.G. of the rivet system is at the geometric center of the rivet system.
2. Primary Shear:

The primary shear force on each rivet is

\[ F_p = \frac{W}{n} = \frac{20000}{4} = 5000 \text{ N}; \]

3. Secondary Shear Force:

\[ W.e = F_{s1}l_1 + F_{s2}l_2 + F_{s3}l_3 + F_{s4}l_4 \]

\[ W.e = wl_1^2 + wl_2^2 + wl_3^2 + wl_4^2 \]

\[ w = \frac{W.e}{l_1^2 + l_2^2 + l_3^2 + l_4^2} = 355.55 \text{ N/mm}; \]

\[ 355.55 \times 150 = 5333.25 \text{ N} \]

\[ \sigma = \frac{5333.25}{\pi d^2/4} \]

\[ 150 = \frac{5333.25}{\pi d^2/4} \]

\[ d^2 = \frac{5333.25 \times 4}{150 \times \pi} \]

\[ d = \sqrt{\frac{21333}{150 \pi}} \]

\[ d = \sqrt{44.7} \approx 6.69 \text{ mm} \]
Design of Boiler Joints

The boiler has a longitudinal joint as well as circumferential joint. The longitudinal joint is used to join the ends of the plate to get the required diameter of a boiler. For this purpose, a butt joint with two cover plates is used. The circumferential joint is used to get the required length of the boiler. For this purpose, a lap joint with one ring overlapping the other alternately is used. Since a boiler is made up of number of rings, therefore the longitudinal joints are staggered for convenience of connecting rings at places where both longitudinal and circumferential joints occur.

Assumptions in Designing Boiler Joints

The following assumptions are made while designing a joint for boilers:
1. The load on the joint is equally shared by all the rivets. The assumption implies that the shell and plate are rigid and that all the deformation of the joint takes place in the rivets themselves.
2. The tensile stress is equally distributed over the section of metal between the rivets.
3. The shearing stress in all the rivets is uniform.
4. The crushing stress is uniform.
5. There is no bending stress in the rivets.
6. The holes into which the rivets are driven do not weaken the member.
7. The rivet fills the hole after it is driven.
8. The friction between the surfaces of the plate is neglected.

Design of Longitudinal Butt Joint for a Boiler

According to Indian Boiler Regulations (I.B.R), the following procedure should be adopted for the design of longitudinal butt joint for a boiler.
1. **Thickness of boiler shell.** First of all, the thickness of the boiler shell is determined by using the thin cylindrical formula, *i.e.*

\[
t = \frac{P \cdot D}{2 \sigma_i \times \eta_i} + 1 \text{ mm as corrosion allowance}
\]

where

- \( t \) = Thickness of the boiler shell,
- \( P \) = Steam pressure in boiler,
- \( D \) = Internal diameter of boiler shell,
- \( \sigma_i \) = Permissible tensile stress, and
- \( \eta_i \) = Efficiency of the longitudinal joint.

2. **Diameter of rivets.** After finding out the thickness of the boiler shell \((t)\), the diameter of the rivet hole \((d)\) may be determined by using Unwin's empirical formula, *i.e.*

\[
d = 6 \sqrt{t}
\]

(when \( t \) is greater than 8 mm)

But if the thickness of plate is less than 8 mm, then the diameter of the rivet hole may be calculated by equating the shearing resistance of the rivets to crushing resistance. In no case, the diameter of rivet hole should not be less than the thickness of the plate, because there will be danger of punch crushing.

3. **Pitch of rivets.** The pitch of the rivets is obtained by equating the tearing resistance of the plate to the shearing resistance of the rivets. It may noted that

- (a) The pitch of the rivets should not be less than \(2d\), which is necessary for the formation of head.
- (b) The maximum value of the pitch of rivets for a longitudinal joint of a boiler as per I.B.R. is

\[
p_{\text{max}} = C \times t + 41.28 \text{ mm}
\]

where

- \( t \) = Thickness of the shell plate in mm, and
- \( C \) = Constant.

4. **Distance between the rows of rivets.** The distance between the rows of rivets as specified by Indian Boiler Regulations is as follows:

- (a) For equal number of rivets in more than one row for lap joint or butt joint, the distance between the rows of rivets \((pb)\) should not be less than \(0.33 \, p + 0.67 \, d\), for zig-zig riveting, and \(2 \, d\), for chain riveting.
- (b) For joints in which the number of rivets in outer rows is half the number of rivets in inner rows and if the inner rows are chain riveted, the distance between the outer rows and the next rows should not be less than \(0.33 \, p + 0.67 \text{ or } 2 \, d\), whichever is greater.

The distance between the rows in which there are full number of rivets shall not be less than \(2d\).

- (c) For joints in which the number of rivets in outer rows is half the number of rivets in inner rows and if the inner rows are zig-zig riveted, the distance between the outer rows and the Next rows shall not be less than \(0.2 \, p + 1.15 \, d\).
The distance between the rows in which there are full number of rivets (zig-zag) shall not be less than \(0.165 p + 0.67 d\).

**Note:** In the above discussion, \(p\) is the pitch of the rivets in the outer rows.

5. **Thickness of butt strap.** According to I.B.R., the thicknesses for butt strap \((t1)\) are as given below:

   (a) The thickness of butt strap, in no case, shall be less than 10 mm.
   \[ t_1 = 1.125 t \]
   \[ t_1 = 1.125 t \left( \frac{p - d}{p - 2d} \right) , \text{for single butt straps, every alternate rivet in outer rows being omitted.} \]
   \[ t_1 = 0.625 t \text{, for double butt-strap of equal width having ordinary riveting (chain riveting).} \]

   (b) For double butt strap of equal width having every alternate rivet in the outer rows being omitted.

6. **Margin.** The margin \((m)\) is taken as 1.5 \(d\).

**Design of Circumferential Lap Joint for a Boiler**

The following procedure is adopted for the design of circumferential lap joint for a boiler.

1. **Thickness of the shell and diameter of rivets.** The thickness of the boiler shell and the diameter of the rivet will be same as for longitudinal joint.

2. **Number of rivets.** Since it is a lap joint, therefore the rivets will be in single shear.

   Shearing resistance of the rivets,
   \[ P_s = n \times \frac{\pi}{4} \times d^2 \times \tau \]

   where \(n = \text{Total number of rivets.}\)

   Knowing the inner diameter of the boiler shell \((D)\), and the pressure of steam \((P)\), the total shearing load acting on the circumferential joint,
   \[ W_s = \frac{\pi}{4} \times D^2 \times P \]

   From equations (i) and (ii), we get
   \[ n \times \frac{\pi}{4} \times d^2 \times \tau = \frac{\pi}{4} \times D^2 \times P \]

   \[ \therefore n = \left( \frac{D}{d} \right)^2 \frac{P}{\tau} \]
3. **Pitch of rivets.** If the efficiency of the longitudinal joint is known, then the efficiency of the circumferential joint may be obtained. It is generally taken as 50% of tearing efficiency in longitudinal joint, but if more than one circumferential joints is used, then it is 62% for the intermediate joints.

Knowing the efficiency of the circumferential lap joint ($\eta_c$), the pitch of the rivets for the lap joint ($p_1$) may be obtained by using the relation:

$$\eta_c = \frac{p_1 - d}{p_1}$$

4. **Number of rows.** The number of rows of rivets for the circumferential joint may be obtained from the following relation:

$$\text{Number of rows} = \frac{\text{Total number of rivets}}{\text{Number of rivets in one row}}$$

and the number of rivets in one row is

$$= \frac{\pi (D + t)}{p_1}$$

where

$$D = \text{Inner diameter of shell.}$$

5. After finding out the number of rows, the type of the joint (i.e. single riveted or double riveted etc.) may be decided. Then the number of rivets in a row and pitch may be re-adjusted. In order to have a leak-proof joint, the pitch for the joint should be checked from Indian Boiler Regulations.

6. The distance between the rows of rivets (i.e. back pitch) is calculated by using the relations as discussed in the previous article.
7. After knowing the distance between the rows of rivets \((p_b)\), the overlap of the plate may be fixed by using the relation,

\[
\text{Overlap} = (\text{No. of rows of rivets} - 1) \ p_b + m
\]

where \(m\) = Margin.

There are several ways of joining the longitudinal joint and the circumferential joint.

Q. A longitudinal joint for a boiler is to be designed for a steam pressure of 2.5 N/mm\(^2\) with an efficiency of 85%. The inside diameter of the largest course of the drum is 1.35 meter. The allowable stresses may be taken as in tension = 78.0 N/mm\(^2\), in shear 62.5 N/mm\(^2\); in crushing = 135.0 N/mm\(^2\).

Solution: Given:

The internal pressure "P" = 2.5 N/mm\(^2\); Efficiency of the joint "\(\eta\)" = 0.85;
The inside diameter of the drum "D" = 1.35 meter;
The allowable stresses may be taken as in tension "\(\sigma_t\)" = 78.0 N/mm\(^2\),

The allowable shear stress "\(\tau\)" = 62.5 N/mm\(^2\);
The allowable crushing stress "\(\sigma_c\)" = 135.0 N/mm\(^2\);
The thickness of the plate \(t = \frac{PD}{2\sigma_t\eta} + 1.0 = \frac{2.5 \times 1350}{2 \times 78.0 \times 0.85} + 1.0 = 26.5\) mm;

THE RIVET HOLE AND RIVET DIAMETER:

\(d = 6.05\sqrt{t} = 6.05 \times \sqrt{26.5} = 31.1\) mm;
Assuming the clearance of 1.5 mm, then the rivet diameter corresponding to the calculated diameter of rivet hole diameter \(d_r = 31.1 - 1.5 = 29.6\) mm;
Standard rivet diameter "\(d_r\)" = 30 mm; \quad \{Ref.: Table # 5.3b Page #68\}
The rivet hole diameter "\(d\)" = 30 + 1.5 = 31.5 mm;

THE PITCH:

Equating the shear and tearing resistance of the joint that is

\[(p - d) \times t \times \sigma_t = 1.875 \times \left(5 \times \frac{\pi}{4} \times d^2\right) \times \tau + \frac{\pi}{4} \times d^2 \times \tau\]

Assuming the joint is triple riveted double butt strap zig-zag riveted joint in which the pitch in the inner rows are half that of the outer row.

\[(p - 31.5) \times 26.5 \times 78.0 = 1.875 \times \left(5 \times \frac{\pi}{4} \times 31.5^2\right) \times 62.5\]

\[p = 252.4132\) mm;
The permissible maximum pitch as per I.B.R

\[p_{\text{max}} = k_1 t + 41\) mm \quad \{Ref.: Equation # 5.23b Page #65\}

\[p_{\text{max}} = 6 \times 26.5 + 41 = 200\) mm

Hence the pitch for the joint is 200 mm.
**BACK PITCH:**

The back pitch between the outer row and the inner row
\[ p_b = 0.2p + 1.15d = 0.2 \times 200 + 1.15 \times 31.5 = 76.225 \approx 76\text{mm}; \quad \text{(Ref.: Equation #5.33a, page #66)} \]

The back pitch between the successive inner rows
\[ p_b = 0.165p + 0.67d = 0.165 \times 200 + 0.67 \times 31.5 = 54.105 \approx 54\text{mm}; \quad \text{(Ref.: Equation #5.33b, page #66)} \]

**THE BUTT STRAP THICKNESS:**

Assuming the unequal width of the straps

Inner strap thickness \( t_i = 0.750t = 0.750 \times 26.5 = 19.875 \approx 20\text{mm}; \quad \text{(Ref.: Equation #5.8b, page #63)} \)

Outer strap thickness \( t_o = 0.625t = 0.625 \times 31.5 = 16.5625 \approx 17\text{mm}; \quad \text{(Ref.: Equation #5.8a, page #63)} \)

**LENGTH OF THE RIVET:**

\[ L = \Sigma t + (1.5 \text{ to } 1.7) \times d \quad \text{(Ref.: Equation #5.9, page #63)} \]

\[ L = \Sigma t + (1.6) \times d = (17 + 26.5 + 20) + 1.6 \times 31.5 = 113.9 \approx 114\text{mm}; \]

**MARGIN:**

\[ m \geq 1.5 \times d \quad \text{(Ref.: Equation #2.34, page #66)} \]

\[ m \geq 1.5 \times 31.5 = 47.25 \text{mm} \]

Taking \( m = 48\text{ mm}; \)

**EFFICIENCY OF THE DESIGNED EFFICIENCY:**

Strength of the unpunched plate = \( \sigma_t \times p \times t = 78.0 \times 200 \times 26.5 = 413400 \text{N}; \)

Tearing resistance of the plate = \( (p - d) \times t \times \sigma_t = (200 - 31.5) \times 26.5 \times 78 = 348289.5 \text{N}; \)

Shearing resistance of the joint = \( 1.875 \times 5 \times \left( \frac{\pi}{4} \times d^2 \right) \times \tau = 1.875 \times 5 \times \left( \frac{\pi}{4} \times 31.5^2 \right) \times 62.5 = 456627.73\text{N}; \)

Crushing resistance of the joint = \( 5 \times d \times t \times \sigma_c = 5 \times 31.5 \times 26.5 \times 135 = 563456.25\text{N} \)

Hence the design efficiency of the joint

\[ \eta = \frac{348289.5}{413400} \times 100 = 84.25\% \]

Even the design efficiency is less than the desired efficiency but this lack in the efficiency is very small as compared to the factor of safety usually taken for the design of longitudinal joint is high i.e. 4 to 5. There for the joint may be assumed to successful.
Unit-III

**Keys:** Key is a mechanical element used on shafts to secure rotating elements like gears, pulleys, or sprockets and prevent relative motion between the two. A key way is a slot or recess in a shaft and a hub of the rotating element to accommodate a key, and the keys used in this way are known as sunk keys. The key transmits torque from the shaft to the shaft supported element or vice versa. It is always inserted parallel to the axis of the shaft.

**Rectangular Key:** It is a sunk key having the rectangular section as shown in below figure. It is used where higher stability is desired in connection. A rectangular key may be tapered, whose width is kept constant while the height is tapered by 1:100. The keyway in hub has the same taper as that in the key while the keyway in the shaft has uniform depth. This key is used for light and medium duty industrial use.

![Rectangular sunk key](image1)

**Figure 3.1**

**Square Keys:** A sunk key having square cross-section as shown in below figure is called square key. The key is sunk half in the shaft and half in the hub. This type of the key is used in light industrial machinery.

![Square sunk key](image2)

**Figure 2.10**

**Splined shafts** are the shaft on which the keys are made integral to it. They are used when there is a relative axial motion between the shaft and the hub. The gear shifting mechanism in automobile gear boxes requires such type of construction. Splines are cut on the shaft by
milling and on the hub by broaching. There are the following three types of the splined shafts:

1. Straight splinted shaft;
2. Involute splined shaft
3. Serrations;

In case of the straight splinted shaft the keys made integral to it are straight. In the involute splined shafts the splines are in the form of concentric external and internal gear teeth having the involute profile. And the serrations are straight splinted shafts but are used in the applications where it is important to keep the overall size of the assembly as small as possible.

![Figure 3.3](Image)

**Figure 3.3**

**INTRODUCTION TO SHAFTS-**

A shaft is a rotating machine element which is used to transmit power from one place to another. The power is delivered to the shaft by some tangential force and the resultant torque (or twisting moment) set up within the shaft permits the power to be transferred to various machines linked up to the shaft. In order to transfer the power from one shaft to another, the various members such as pulleys, gears etc., are mounted on it. These members along with the forces exerted upon them causes the shaft to bending. In other words, we may say that a shaft is used for the transmission of torque and bending moment. The various members are mounted on the shaft by means of keys or splines.

**Notes:** 1. The shafts are usually cylindrical, but may be square or cross-shaped in section. They are solid in cross-section but sometimes hollow shafts are also used.

2. An **axle**, though similar in shape to the shaft, is a stationary machine element and is used for the transmission of bending moment only. It simply acts as a support for some rotating
body such as hoisting drum, a car wheel or a rope sheave.

3. A **spindle** is a short shaft that imparts motion either to a cutting tool (e.g. drill press spindles) or to a work piece (e.g. lathe spindles).

**Material Used for Shafts**
The material used for shafts should have the following properties:
1. It should have high strength.
2. It should have good machinability.
3. It should have low notch sensitivity factor.
4. It should have good heat treatment properties.
5. It should have high wear resistant properties.

The material used for ordinary shafts is carbon steel of grades 40 C 8, 45 C 8, 50 C 4 and 50 C 12.

The mechanical properties of these grades of carbon steel are given in the following table.

**Mechanical properties of steels used for shafts**

<table>
<thead>
<tr>
<th>Indian standard designation</th>
<th>Ultimate tensile strength, MPa</th>
<th>Yield strength, MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>40 C 8</td>
<td>560 - 670</td>
<td>320</td>
</tr>
<tr>
<td>45 C 8</td>
<td>610 - 700</td>
<td>350</td>
</tr>
<tr>
<td>50 C 4</td>
<td>640 - 760</td>
<td>370</td>
</tr>
<tr>
<td>50 C 12</td>
<td>700 Min.</td>
<td>390</td>
</tr>
</tbody>
</table>

When a shaft of high strength is required, then an alloy steel such as nickel, nickel-chromium or chrome-vanadium steel is used.

**Types of Shafts**
The following two types of shafts are important from the subject point of view:

1. **Transmission shafts.** These shafts transmit power between the source and the machines absorbing power. The counter shafts, line shafts, over head shafts and all factory shafts are transmission shafts. Since these shafts carry machine parts such as pulleys, gears etc., therefore they are subjected to bending in addition to twisting.

2. **Machine shafts.** These shafts form an integral part of the machine itself. The crank shaft is an example of machine shaft.

**Standard Sizes of Transmission Shafts**
The standard sizes of transmission shafts are:
25 mm to 60 mm with 5 mm steps; 60 mm to 110 mm with 10 mm steps; 110 mm to 140 mm with 15 mm steps; and 140 mm to 500 mm with 20 mm steps.
The standard length of the shafts are 5 m, 6 m and 7 m.
Stresses in Shafts
The following stresses are induced in the shafts:
1. Shear stresses due to the transmission of torque (i.e. due to torsional load).
2. Bending stresses (tensile or compressive) due to the forces acting upon machine elements like gears, pulleys etc. as well as due to the weight of the shaft itself.
3. Stresses due to combined torsional and bending loads.

Design of Shafts
The shafts may be designed on the basis of
1. Strength, and
2. Rigidity and stiffness.

In designing shafts on the basis of strength, the following cases may be considered:
(a) Shafts subjected to twisting moment or torque only,
(b) Shafts subjected to bending moment only,
(c) Shafts subjected to combined twisting and bending moments, and
(d) Shafts subjected to axial loads in addition to combined torsional and bending loads.

Shafts Subjected to Twisting Moment Only
When the shaft is subjected to a twisting moment (or torque) only, then the diameter of the shaft may be obtained by using the torsion equation. We know that
\[ \frac{T}{J} = \frac{\tau}{r} \]  \( \ldots (i) \)

where \( T \) = Twisting moment (or torque) acting upon the shaft,
\( J \) = Polar moment of inertia of the shaft about the axis of rotation,
\( \tau \) = Torsional shear stress, and
\( r \) = Distance from neutral axis to the outer most fibre

\( = d/2 \); where \( d \) is the diameter of the shaft.

We know that for round solid shaft, polar moment of inertia,
\[ J = \frac{\pi}{32} \times d^4 \]

The equation \((i)\) may now be written as
\[ \frac{T}{\frac{\pi}{32} \times d^4} = \frac{\tau}{\frac{d}{2}} \quad \text{or} \quad T = \frac{\pi}{16} \times \tau \times d^3 \]  \( \ldots (ii) \)

From this equation, we may determine the diameter of round solid shaft \( (d) \).

We also know that for hollow shaft, polar moment of inertia,
\[ J = \frac{\pi}{32} \left[ (d_o)^4 - (d_i)^4 \right] \]
where $d_o$ and $d_i$ = Outside and inside diameter of the shaft, and $r = d_o / 2$.

Substituting these values in equation (i), we have

$$\frac{T}{\frac{\pi}{32} [(d_o)^4 - (d_i)^4]} = \frac{\tau}{\frac{d_o}{2}} \quad \text{or} \quad T = \frac{\pi}{16} \times \tau \left[ \frac{(d_o)^4 - (d_i)^4}{d_o} \right]$$

... (iii)

Let $k = \text{Ratio of inside diameter and outside diameter of the shaft} = \frac{d_i}{d_o}$

Now the equation (iii) may be written as

$$T = \frac{\pi}{16} \times \tau \times \frac{(d_o)^4}{d_o} \left[ 1 - \left( \frac{d_i}{d_o} \right)^4 \right] = \frac{\pi}{16} \times \tau \left( d_o \right)^3 \left( 1 - k^4 \right)$$

... (iv)

From the equations (iii) or (iv), the outside and inside diameter of a hollow shaft may be determined.

It may be noted that

1. The hollow shafts are usually used in marine work. These shafts are stronger per kg of material and they may be forged on a mandrel, thus making the material more homogeneous than would be possible for a solid shaft.

When a hollow shaft is to be made equal in strength to a solid shaft, the twisting moment of both the shafts must be same. In other words, for the same material of both the shafts,

$$T = \frac{\pi}{16} \times \tau \left[ \frac{(d_o)^4 - (d_i)^4}{d_o} \right] = \frac{\pi}{16} \times \tau \times d^3$$

$$\therefore \quad \frac{(d_o)^4 - (d_i)^4}{d_o} = d^3 \quad \text{or} \quad (d_o)^3 \left( 1 - k^4 \right) = d^3$$

2. The twisting moment ($T$) may be obtained by using the following relation:

We know that the power transmitted (in watts) by the shaft,

$$P = \frac{2 \pi N \times T}{60} \quad \text{or} \quad T = \frac{P \times 60}{2 \pi N}$$

where $T =$ Twisting moment in N-m, and

$N =$ Speed of the shaft in r.p.m.

3. In case of belt drives, the twisting moment ($T$) is given by

$$T = (T_1 - T_2) R$$

where $T_1$ and $T_2 =$ Tensions in the tight side and slack side of the belt respectively, and

$R =$ Radius of the pulley.

Shafts Subjected to Bending Moment Only
When the shaft is subjected to a bending moment only, then the maximum stress (tensile or compressive) is given by the bending equation. We know that

\[
\frac{M}{I} = \frac{\sigma_b}{y} \quad \ldots (i)
\]

where \(M\) = Bending moment,
\(I\) = Moment of inertia of cross-sectional area of the shaft about the axis of rotation.
\(\sigma_b\) = Bending stress, and
\(y\) = Distance from neutral axis to the outer-most fibre.

We know that for a round solid shaft, moment of inertia,

\[
I = \frac{\pi}{64} \times d^4 \quad \text{and} \quad y = \frac{d}{2}
\]

Substituting these values in equation (i), we have

\[
\frac{M}{\frac{\pi}{64} \times d^4} = \frac{\sigma_b}{\frac{d}{2}} \quad \text{or} \quad M = \frac{\pi}{32} \times \sigma_b \times d^3
\]

From this equation, diameter of the solid shaft \((d)\) may be obtained.

We also know that for a hollow shaft, moment of inertia,

\[
I = \frac{\pi}{64} \left[ (d_o)^4 - (d_i)^4 \right] = \frac{\pi}{64} (d_o)^4 \left( 1 - k^4 \right) \quad \ldots \text{(where} \ k = \frac{d_i}{d_o} \text{)}
\]

and

\[
y = \frac{d_o}{2}
\]

Again substituting these values in equation (i), we have

\[
\frac{M}{\frac{\pi}{64} (d_o)^4 \left( 1 - k^4 \right)} = \frac{\sigma_b}{\frac{d_o}{2}} \quad \text{or} \quad M = \frac{\pi}{32} \times \sigma_b \left( d_o \right)^3 \left( 1 - k^4 \right)
\]

From this equation, the outside diameter of the shaft \((d_o)\) may be obtained.

**Shafts Subjected to Combined Twisting Moment and Bending Moment**

When the shaft is subjected to combined twisting moment and bending moment, then the shaft must be designed on the basis of the two moments simultaneously. Various theories have been suggested to account for the elastic failure of the materials when they are subjected to various types of combined stresses. The following two theories are important from the subject point of view:

1. Maximum shear stress theory or Guest's theory. It is used for ductile materials such as mild steel.
2. Maximum normal stress theory or Rankine’s theory. It is used for brittle materials such as cast iron.

Let \(\tau\) = Shear stress induced due to twisting moment, and
σ_b = Bending stress (tensile or compressive) induced due to bending moment.

According to maximum shear stress theory, the maximum shear stress in the shaft,

\[ \tau_{\text{max}} = \frac{1}{2} \sqrt{(\sigma_b)^2 + 4 \tau^2} \]

Substituting the values of \( \tau \) and \( \sigma_b \)

\[ \tau_{\text{max}} = \frac{1}{2} \sqrt{\left( \frac{32M}{\pi d^3} \right)^2 + 4 \left( \frac{16T}{\pi d^3} \right)^2} = \frac{16}{\pi d^3} \sqrt{M^2 + T^2} \]

or

\[ \frac{\pi}{16} \times \tau_{\text{max}} \times d^3 = \sqrt{M^2 + T^2} \]

The expression \( \sqrt{M^2 + T^2} \) is known as equivalent twisting moment and is denoted by \( Te \). The equivalent twisting moment may be defined as that twisting moment, which when acting alone, produces the same shear stress (\( \tau \)) as the actual twisting moment. By limiting the maximum shear stress (\( \tau_{\text{max}} \)) equal to the allowable shear stress (\( \tau \)) for the material, the equation (i) may be written as

\[ T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} \times \tau \times d^3 \]

...(ii)

From this expression, diameter of the shaft (\( d \)) may be evaluated.

Now according to maximum normal stress theory, the maximum normal stress in the shaft,

\[ \sigma_{b(\text{max})} = \frac{1}{2} \sigma_b + \frac{1}{2} \sqrt{(\sigma_b)^2 + 4 \tau^2} \]

...(iii)

\[ = \frac{1}{2} \times \frac{32M}{\pi d^3} + \frac{1}{2} \sqrt{\left( \frac{32M}{\pi d^3} \right)^2 + 4 \left( \frac{16T}{\pi d^3} \right)^2} \]

\[ = \frac{32}{\pi d^3} \left[ \frac{1}{2} (M + \sqrt{M^2 + T^2}) \right] \]

or

\[ \frac{\pi}{32} \times \sigma_{b(\text{max})} \times d^3 = \frac{1}{2} \left[ M + \sqrt{M^2 + T^2} \right] \]

...(iv)

The expression

\[ \frac{1}{2} \left[ (M + \sqrt{M^2 + T^2}) \right] \]

is known as equivalent bending moment and is denoted by \( M_e \). The equivalent bending moment may be defined as that moment which when acting alone produces the same
tensile or compressive stress \((\sigma_b)\) as the actual bending moment. By limiting the maximum normal stress \([\sigma_b(\text{max})]\) equal to the allowable bending stress \((\sigma_b)\), then the equation (iv) may be written as

\[ M_e = \frac{1}{2} \left[ M + \sqrt{M^2 + T^2} \right] = \frac{\pi}{32} \times \sigma_b \times d^3 \]  

...(v)

From this expression, diameter of the shaft \((d)\) may be evaluated.

Notes: 1. In case of a hollow shaft, the equations (ii) and (v) may be written as

\[ T_e = \frac{1}{16} \times \sigma (d_o)^3 (1 - k^4) \]

and

\[ M_e = \frac{1}{2} \left[ M + \sqrt{M^2 + T^2} \right] = \frac{\pi}{32} \times \sigma_b (d_o)^3 (1 - k^4) \]

2. It is suggested that diameter of the shaft may be obtained by using both the theories and the larger of the two values is adopted.

Shafts Subjected to Axial Load in addition to Combined Torsion and Bending Loads

When the shaft is subjected to an axial load \((F)\) in addition to torsion and bending loads as in propeller shafts of ships and shafts for driving worm gears, then the stress due to axial load must be added to the bending stress \((\sigma_b)\). We know that bending equation is

\[ \frac{M}{I} = \frac{\sigma_b}{y} \]  

or  \[ \sigma_b = \frac{M}{I} \times \frac{d}{2} = \frac{32 M}{\pi d^3} \]

and stress due to axial load

\[ = \frac{F}{\frac{\pi}{4} \times d^2} = \frac{4 F}{\pi d^2} \]  

...(For round solid shaft)

\[ = \frac{F}{\frac{\pi}{4} \left[ (d_o)^2 - (d_i)^2 \right]} = \frac{4 F}{\pi \left[ (d_o)^2 - (d_i)^2 \right]} \]

...(For hollow shaft)

\[ = \frac{F}{\pi (d_o)^2 \left( 1 - k^2 \right)} \]  

\[ \because k = d/d_o \]

Resultant stress (tensile or compressive) for solid shaft,

\[ \sigma_1 = \frac{32 M}{\pi d^3} + \frac{4 F}{\pi d^2} = \frac{32}{\pi d^3} \left( M + \frac{F \times d}{8} \right) \]  

...(i)

\[ = \frac{32 M_1}{\pi d^3} \]  

\[ \because \text{Substituting } M_1 = M + \frac{F \times d}{8} \]
In case of a hollow shaft, the resultant stress,

\[ \sigma_1 = \frac{32 M}{\pi (d_o)^3 (1 - k^4)} + \frac{4 F}{\pi (d_o)^2 (1 - k^4)} \]

\[ = \frac{32}{\pi (d_o)^3 (1 - k^4)} \left[ M + \frac{F d_o (1 + k^2)}{8} \right] = \frac{32 M_1}{\pi (d_o)^3 (1 - k^4)} \]

... [Substituting for hollow shaft, \( M_1 = M + \frac{F d_o (1 + k^2)}{8} \)]

In case of long shafts (slender shafts) subjected to compressive loads, a factor known as **column factor** (\( \alpha \)) must be introduced to take the column effect into account.

\[ \therefore \text{Stress due to the compressive load,} \]

\[ \sigma_c = \frac{\alpha \times 4 F}{\pi d^2} \]

\[ = \frac{\alpha \times 4 F}{\pi (d_o)^2 (1 - k^2)} \]

... [For hollow shaft]

\[ \text{The value of column factor (\( \alpha \)) for compressive loads* may be obtained from the following relation:} \]

\[ \text{Column factor,} \quad \alpha = \frac{1}{1 - 0.0044 (L/K)} \]

This expression is used when the slenderness ratio \( (L/K) \) is less than 115. When the slenderness ratio \( (L/K) \) is more than 115, then the value of column factor may be obtained from the following relation:

\[ \text{Column factor,} \quad \alpha = \frac{\sigma_y (L/K)^2}{C \pi^2 E} \]

where \( L \) = Length of shaft between the bearings,

\( K \) = Least radius of gyration,

\( \sigma_y \) = Compressive yield point stress of shaft material, and

\( C \) = Coefficient in Euler's formula depending upon the end conditions.

The following are the different values of \( C \) depending upon the end conditions.

- \( C = 1 \), for hinged ends,
- \( = 2.25 \), for fixed ends,
- \( = 1.6 \), for ends that are partly restrained as in bearings.

**Note:** In general, for a hollow shaft subjected to fluctuating torsional and bending load, along with an axial load, the equations for equivalent twisting moment \( (T_e) \) and equivalent bending moment \( (M_e) \)
It may be noted that for a solid shaft, \( k = 0 \) and \( d_0 = d \). When the shaft carries no axial load, then \( F = 0 \) and when the shaft carries axial tensile load, then \( \alpha = 1 \).

**Design of Shafts on the basis of Rigidity**

Sometimes the shafts are to be designed on the basis of rigidity. We shall consider the following two types of rigidity.

1. **Torsional rigidity.** The torsional rigidity is important in the case of camshaft of an I.C. engine where the timing of the valves would be affected. The permissible amount of twist should not exceed 0.25° per metre length of such shafts. For line shafts or transmission shafts, deflections 2.5 to 3 degree per metre length may be used as limiting value. The widely used deflection for the shafts is limited to 1 degree in a length equal to twenty times the diameter of the shaft. The torsional deflection may be obtained by using the torsion equation,

\[
\theta = \frac{T}{J} = \frac{G \cdot \theta}{L} \text{ or } \theta = \frac{T \cdot L}{J \cdot G}
\]

where \( \theta \) = Torsional deflection or angle of twist in radians, 
\( T \) = Twisting moment or torque on the shaft, 
\( J \) = Polar moment of inertia of the cross-sectional area about the axis of rotation,

\[
\begin{align*}
&= \frac{\pi}{32} \times d^4 \quad \text{...(For solid shaft)} \\
&= \frac{\pi}{32} \left[ (d_0)^4 - (d_i)^4 \right] \quad \text{...(For hollow shaft)}
\end{align*}
\]

\( G \) = Modulus of rigidity for the shaft material, and 
\( L \) = Length of the shaft.

2. **Lateral rigidity.** It is important in case of transmission shafting and shafts running at high speed, where small lateral deflection would cause huge out-of-balance forces. The lateral rigidity is also important for maintaining proper bearing clearances and for correct gear teeth alignment. If the shaft is of uniform cross-section, then the lateral deflection of a shaft may be obtained by using the deflection formulae as in Strength of Materials. But when the shaft is of variable cross-section, then the lateral deflection may be determined from the fundamental equation for the elastic curve of a beam, \( i.e. \)
COUPLINGS

Couplings are used to connect two shafts for torque transmission in varied applications. It may be to connect two units such as a motor and a generator or it may be to form a long line shaft by connecting shafts of standard lengths say 6-8m by couplings. Coupling may be rigid or they may provide flexibility and compensate for misalignment. They may also reduce shock loading and vibration. A wide variety of commercial shaft couplings are available ranging from a simple keyed coupling to one which requires a complex design procedure using gears or fluid drives etc. However there are two main types of couplings:

- Rigid couplings
- Flexible couplings

Rigid couplings are used for shafts having no misalignment while the flexible couplings can absorb some amount of misalignment in the shafts to be connected. In the next section we shall discuss different types of couplings and their uses under these two broad headings.

ADVANTAGES OF COUPLINGS:

1. It provides the connection of shafts of two different units such as an electric motor and a machine.
2. It makes the provision for disconnection of two units for repair or alteration.
3. It introduces the mechanical flexibility between two connected units.
4. It reduces the transmission of vibrations and shocks between two connected units.
5. It makes the maintenance easy.

REQUIREMENTS OF A GOOD COUPLING:

1. It should transmit full power from one shaft to another.
2. It should keep the shaft in perfect alignment.
3. It should absorb the slight misalignment that may present between the driver and driven shaft.
4. It should be easy to connect and disconnect.
5. It should have no projecting parts.

Rigid Flange Coupling

A typical rigid flange coupling is shown in following figure.
Figure 3.4

It essentially consists of two cast iron flanges which are keyed to the shafts to be joined. The flanges are brought together and are bolted in the annular space between the hub and the protecting flange. The protective flange is provided to guard the projecting bolt heads and nuts. The bolts are placed equi-spaced on a bolt circle diameter and the number of bolt depends on the shaft diameter $d$. A spigot ‘A’ on one flange and a recess on the opposing face is provided for ease of assembly.

The design procedure is generally based on determining the shaft diameter $d$ for a given torque transmission and then following empirical relations different dimensions of the coupling are obtained. Check for different failure modes can then be carried out. Design details of such couplings will be discussed in the next lesson. The main features of the design are essentially

- (a) Design of bolts
- (b) Design of hub
- (c) Overall design and dimensions.

Design procedure is given in the following steps:

1. Diameter ‘$d$’ based on torque transmission is given by
   \[ d = \left( \frac{16T}{\pi \tau_y} \right)^{1/3} \]

   Where $T$ is the torque and $\tau_y$ is the yield stress in shear.

2. Hub diameter $d_1 = 1.75d + 6.5\text{mm}$

3. Hub length $L = 1.5d$

   But the hub length also depends on the length of the key. Therefore this length $L$ must be checked while finding the key dimension based on shear and crushing failure modes.

4. Key dimensions:
If a square key of sides $b$ is used then $b$ is commonly taken as $d/4$. In case, for shear failure we have

$$\left(\frac{d}{4}L_k\right) \cdot \tau_y \cdot \frac{d}{2} = T$$

where $L_k$ is the length of the key and $\tau_y$ is the shear stress.

If $L_k$ determined here is less than hub length $L$ we may assume the key length to be the same as hub length.

For Crushing

$$\left(\frac{d}{8}L_k\right) \sigma_c \cdot \frac{d}{2} = T \text{ where } \sigma_c \text{ is crushing stress induced in the key. This gives}$$

$$\sigma_c = \frac{16T}{L_k d^2}$$

and if $\sigma_c < \sigma_{cy}$, the bearing strength of the key material, the key dimensions chosen are in order.

(5) Bolt dimensions:

The bolts are subjected to shear and bearing stresses while transmitting torque. Considering the shear failure mode we have

$$n \cdot \frac{\pi}{4} d_b^2 \tau_{yb} \cdot \frac{d_c}{2} = T$$

where $n$ is the number of bolts, $d_b$ the nominal bolt diameter, $T$ is the torque transmitted, $\tau_{yb}$ is the shear yield strength of the bolt material and $d_c$ is the bolt circle diameter. The bolt diameter may now be obtained if $n$ is known.

The number of bolts $n$ is often given by the following empirical relation:

$$n = \frac{4}{150} d + 3$$

Where $d$ is the shaft diameter in mm. The bolt circle diameter must be such that it should provide clearance for socket wrench to be used for the bolts. The empirical relation takes care of this.

Considering crushing failure we have

$$n \cdot d_b \cdot t_2 \sigma_{cyb} \cdot \frac{d_c}{2} = T$$

where $t_2$ is the flange width over which the bolts make contact and $\sigma_{cyb}$ is the yield crushing strength of the bolt material. This gives $t_2$. Clearly the bolt length must be more than $2t_2$ and a suitable standard length for the bolt diameter may be chosen from handbook.
(6) A protecting flange is provided as a guard for bolt heads and nuts. The thickness $t_3$ is less than $t_2/2$. The corners of the flanges should be rounded.

(7) The spigot depth is usually taken between 2-3mm.

(8) Another check for the shear failure of the hub is to be carried out. For this failure mode we may write

$$\pi d_1 t_2 \tau_{yf} \frac{d_1}{2} = T$$

where $d_1$ is the hub diameter and $\tau_{yf}$ is the shear yield strength of the flange material. Knowing $\tau_{yf}$ we may check if the chosen value of $t_2$ is satisfactory or not. Finally, knowing hub diameter $d_1$, bolt diameter and protective thickness $t_2$ we may decide the overall diameter $d_3$.

Flexible rubber – bushed couplings

This is simplest type of flexible coupling and a typical coupling of this type is shown in Figure.

![Flexible rubber – bushed couplings](image)

Figure: 3.4 A typical flexible coupling with rubber bushings.

In a rigid coupling the torque is transmitted from one half of the coupling to the other through the bolts and in this arrangement shafts need be aligned very well. However in the bushed coupling the rubber bushings over the pins (bolts) (as shown in Figure) provide flexibility and these coupling can accommodate some misalignment. Because of the rubber bushing the design for pins should be considered carefully.
(1) Bearing stress

Rubber bushings are available for different inside and outside diameters. However rubber bushes are mostly available in thickness between 6 mm to 7.5 mm for bores up to 25 mm and 9 mm thickness for larger bores. Brass sleeves are made to suit the requirements. However, brass sleeve thickness may be taken to be 1.5 mm. The outside diameter of rubber bushing $d_r$ is given by

$$d_r = d_b + 2t_{br} + 2t_r$$

where $d_b$ is the diameter of the bolt or pin, $t_{br}$ is the thickness of the brass sleeve and $t_r$ is the thickness of rubber bushing. We may now write

$$n.d_r t_{2p_b} \frac{d_c}{2} = T$$

where $d_c$ is the bolt circle diameter and $t_2$ the flange thickness over the bush contact area. A suitable bearing pressure for rubber is 0.035 N/mm$^2$

the number of pin is given by

$$n = \frac{d}{25} + 3$$

The $d_c$ here is different from what we had for rigid flange bearings. This must be judged considering the hub diameters, outside diameter of the bush and a suitable clearance. A rough drawing is often useful in this regard.

![Diagram](image)

### Loading on a pin supporting the bushings.

Clearly the bearing pressure that acts as distributed load on rubber bush would produce bending of the pin. Considering an equivalent concentrated load $F = pt_2d$ the bending stress is

$$\sigma_b = \frac{32F(t_2/2)}{\pi d_2^3}$$

Knowing the shear and bending stresses we may check the pin diameter for principal stresses using appropriate theories of failure.
We may also assume the following empirical relations:

Hub diameter = 2d  
Hub length = 1.5d  
\( \frac{0.5d}{\sqrt{n}} \)  
Pin diameter at the neck = 

**Compare The Rigid Flange Coupling And Bush Pin Type Flexible Coupling.**

<table>
<thead>
<tr>
<th>Rigid Flange Coupling</th>
<th>Bush Pin Type Flexible Coupling</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Rigid flange coupling can not bear any kind of misalignment.</td>
<td>1. Bush pin type flexible coupling can bear angular misalignment up to 0.50° and the lateral misalignment up to maximum 5 mm.</td>
</tr>
<tr>
<td>2. Rigid flange coupling has smaller axial dimensions.</td>
<td>2. Bush pin type flexible coupling has larger axial dimensions as compared to rigid flange coupling.</td>
</tr>
<tr>
<td>3. Both flanges are of equal thickness</td>
<td>3. Input flange is of larger thickness as compared to the output flange.</td>
</tr>
<tr>
<td>4. Bolts are used to connect the two flanges of the rigid flange coupling</td>
<td>4. Pin bush assembly is used for connecting the input and output flanges.</td>
</tr>
<tr>
<td>5. Torque can be transmitted from input flange to the output flange by shear strength of the bolts as well as through the friction between the flanges.</td>
<td>5. Torque can be transmitted from input flange to the output flange by shear strength of the pin and bearing stress of the rubber bush as well as through the friction between the flanges.</td>
</tr>
<tr>
<td>6. Two flanges touches each other except in the recess provided in the hub portion to assure the perfect alignment of both the shafts.</td>
<td>6. There is always a gap of up to 5 mm between the two flanges to bear the angular misalignment.</td>
</tr>
<tr>
<td>7. Rigid flange coupling is cheaper</td>
<td>7. Bush pin type flexible coupling is costlier as compared to rigid flange coupling.</td>
</tr>
</tbody>
</table>

Q. **It is required to design a square key for fixing a pulley on the shaft, which is 50 mm in diameter. The pulley transmits 10 kW power at 200 rpm to the shaft. The key is made of steel 45C8 (\( \sigma_{yt} = \sigma_{yc} = 380 \text{ MPa} \)) and the factor of safety is 3. Determine the dimensions of the key. Assume (\( \sigma_{sy} = 0.577 \sigma_{yt} \)).**
Solution: The torque is to be transmitted

\[ T = \frac{kW \times 60 \times 10^6}{2 \times \pi \times N} \]
\[ T = \frac{10 \times 60 \times 10^6}{2 \times \pi \times N} = 477464.83 \text{ N-mm}; \]

Permissible Stresses:

- **Permissible crushing stress**
  \[ \sigma_{cp} = \frac{\sigma_{vc}}{f.o.s} = \frac{380}{3} = 126.66 \text{ MPa}; \]

- **Permissible shear stress**
  \[ \tau_p = \frac{\sigma_{ys}}{f.o.s} = \frac{0.577 \times \sigma_{ys}}{3} = \frac{0.577 \times 380}{3} = 73.08 \text{ MPa}; \]

Cross section dimensions of the keys:

Refer Table no. 4.1 page no. 60, Design data handbook for mechanical engineers by K. Mahadevan and K. Balveera Reddy;

The cross section dimensions corresponding to the shaft diameter 50 mm are as follows:
- Width of the key “b” = 16 mm
- Height of the key “h” = 10 mm

Taking the dimension for square key b = 16 mm and h=16 mm

**Now determining the length of the key “L”:**

Against crushing:

\[ \sigma_{cp} \geq \frac{4 \times T}{d \times h \times l}; \]
\[ 126.66 \geq \frac{4 \times 477464.83}{50 \times 16 \times l}; \]
\[ l \geq \frac{4 \times 477464.83}{50 \times 16 \times 126.66}; \]
\[ l \geq 18.85 \text{ mm}; \]

Against shearing:

\[ \tau_p \geq \frac{2 \times T}{d \times b \times l}; \]
\[ 73.08 \geq \frac{2 \times 477464.83}{50 \times 16 \times l}; \]
\[ l \geq \frac{2 \times 477464.83}{50 \times 16 \times 73.08}; \]
\[ l \geq 16.33 \text{ mm}; \]

The minimum length of the key should be 18.85 mm. Taking it as 20 mm. Hence the dimensions of the key are
- b = 16 mm;
- h = 16 mm;
- l = 20 mm.
Q. A mild steel shaft transmits 20 kW at 200 rpm. It is subjected to a bending moment of 562.5 N-m. Determine the size of the shaft, if the allowable shear stress is 42 MPa, and the maximum tensile or compressive stress is not to exceed 58 MPa. What size of the shaft will be required if it is subjected to gradually applied load.

Solution:

Given:

Power: \( P = 20 \text{ kW} \), Rotational speed \( N = 200 \text{ rpm} \); Bending moment \( M = 562.5 \text{ N-m} \);

Allowable shear stress \( \tau_{\text{all}} = 42 \text{ MPa} \); Normal stress \( \sigma_{\text{all}} = 58 \text{ MPa} \)

Torque:

\[
T = \frac{kW \times 10^6 \times 60}{2\pi N} = \frac{20 \times 10^6 \times 60}{2 \times \pi \times 200} = 954929.6585 \text{ N-mm};
\]

Equivalent Bending Moment:

\[
M_e = \frac{1}{2} \left[ M + \sqrt{M^2 + T^2} \right] = \frac{1}{2} \left[ 562500 + \sqrt{562500^2 + 954929.6585^2} \right]
\]

\[
M_e = \frac{1}{2} \left[ 562500 + 1108285.569 \right] = 835392.7845 \text{ N-mm}
\]

Equivalent Twisting Moment:

\[
T_e = \sqrt{M^2 + T^2} = \sqrt{562500^2 + 954929.6585^2} = 1108285.569 \text{ N-mm};
\]

The maximum shear stress induced in the material of the shaft

\[
\tau_{\text{max}} = \frac{16T_e}{\pi d^3} = \frac{16 \times 1108285.569}{\pi d^3} = \frac{5644452.053}{d^3}
\]

Hence using the maximum shear stress theory

\[
42 \geq \frac{5644452.053}{d^3} \Rightarrow d^3 \geq \frac{5644452.053}{42}
\]

\[
d^3 \geq 134391.7156
\]

\[
d \geq 51.22 \text{ mm};
\]

The maximum normal (tensile or compressive) stress induced in the material of the shaft

\[
\sigma_{\text{max}} = \frac{32M_e}{\pi d^3} = \frac{32 \times 835392.7845}{\pi d^3} = \frac{8509241.029}{d^3}
\]

Hence using the maximum normal stress theory

\[
58 \geq \frac{8509241.029}{d^3} \Rightarrow d^3 \geq \frac{8509241.029}{58}
\]

\[
d^3 \geq 146711.0522
\]

\[
d \geq 52.74 \text{ mm};
\]

Considering the gradual application of the load:

Assuming:

- shock and fatigue factor for bending and torsion \( K_t = 1.0 \)
- Then shock and fatigue factor for bending \( K_b = 1.5 \)
Equivalent Bending Moment:

\[ M_e = \frac{1}{2} \left[ K_h M + \sqrt{(K_h M)^2 + (K_c T)^2} \right] = \frac{1}{2} \left[ 1.5 \times 562500 + \sqrt{(1.5 \times 562500)^2 + (1 \times 954929.6585)^2} \right] \]

\[ M_e = 1059016.483 \text{ N-mm} \]

Equivalent Twisting Moment:

\[ T_e = \sqrt{(K_h M)^2 + (K_c T)^2} = \sqrt{(1.5 \times 562500)^2 + (1 \times 954929.6585)^2} \]

\[ T_e = 1274285.963 \text{ N-mm} \]

The maximum shear stress induced in the material of the shaft

\[ \tau_{\text{max}} = \frac{16 T_e}{\pi d^3} = \frac{16 \times 1274285.963}{\pi d^3} = \frac{6489885.117}{d^3} \]

Hence using the maximum shear stress theory

\[ 42 \geq \frac{6489885.117}{d^3} \Rightarrow d^3 \geq \frac{6489885.117}{42} \]

\[ d^3 \geq 154521.0742 \]

\[ d \geq 53.66 \text{ mm}; \]

The maximum normal (tensile or compressive) stress induced in the material of the shaft

\[ \sigma_{\text{max}} = \frac{32 M_e}{\pi d^3} = \frac{32 \times 1059016.483}{\pi d^3} = \frac{10787053.32}{d^3} \]

Hence using the maximum normal stress theory

\[ 58 \geq \frac{10787053.32}{d^3} \Rightarrow d^3 \geq \frac{10787053.32}{58} \]

\[ d^3 \geq 185983.6779 \]

\[ d \geq 57.08 \text{ mm}; \]

Hence the diameter of the shaft is 57.08 mm or 58 mm.

Q. A rigid flange coupling is used to transmit a torque of 215 N-m from a shaft of 35 mm diameter to another shaft of the same diameter. Design the hub, flange thickness and the bolt diameter on the basis of shear strength of the bolt material. Assuming the permissible stresses as follows: Shear stress for the material of the bolt = 40 MPa; Crushing stress for the material of the bolt = 80 MPa; Shear stress for the cast iron = 8 MPa.

Solution:

Given: Torque "T" = 215 N-m;
Shaft diameter "d" = 35 mm; Bending moment M = 562.5 N-m; Alloable shear stress cast iron \( \tau_{ci} = 8 \text{MPa} \); Alloable shear stress for the bolt material \( \tau_{ba} = 40 \text{MPa} \); Allowable stress for the bolt material \( \sigma_{ba} = 80 \text{MPa} \);

Design of the HUB:
The dimensions of the hub of the rigid flange coupling emperically may be given as:
Inner diameter of hub \( D_{hi} \) = \( d = 35 \text{mm} \);
Outer diameter of hub \( D_{ho} \) = \( 2d = 70 \text{mm} \);
length of the hub \( l_h = 1.5d = 52.5 \text{mm} \);
\[ k = \frac{D_{hi}}{D_{ho}} = \frac{d}{2d} = 0.5 \]

The induced torsional shear stress in the material of the hub may be given as
\[ \tau_h = \frac{16T}{\pi \times D_{ho}^3 \times (1 - k^4)} = \frac{16 \times 215 \times 10^3}{\pi \times 70^3 \times (1 - 0.5^4)} = 3.405 \text{ N/mm}^2 \]
Since \( \tau_h < 8 \text{ N/mm}^2 \) (i.e permissible shear stress of the cast iron) hence the dimensions taken for the hub are safe.

**Design of Flange Thickness:**

The flange thickness of the rigid flange coupling emperically may be given as:
\( t_f = 0.5d = 17.5 \text{mm} \);
The induced torsional shear stress in the material of the flange at the junction with hub may be given as
\[ \tau_f = \frac{2T}{\pi \times D_{ho}^2 \times t_f} = \frac{2 \times 215 \times 10^3}{\pi \times 70^2 \times 17.5} = 1.596 \approx 1.6 \text{ N/mm}^2 \]
Since \( \tau_f < 8 \text{ N/mm}^2 \) (i.e permissible shear stress of the cast iron) hence the dimensions taken for the flange are safe.

**Design of Bolt:**

Assuming that the bolts are fitted in the reamed holes, hence power is to be transmitted by the shear strength of the bolt material.

Number of the bolts used in the coupling \( "n" = 4 \) \{Assumed\}

\[ T = \left[ \left( \frac{\pi}{4} d_b^2 \right) \times \tau_b \times n \right] \times \frac{D_b}{2} \]

Where
\( d_b = \) nominal diameter of the bolt in \text{mm};
\( D_b = \) Bolt Circle Diameter = \( 3d = 3 \times 35 = 105 \text{mm} \); &
\( \tau_b = \) permissible shear stress of the bolt material.
\[ d_b = \sqrt[3]{\frac{8T}{\pi \times \tau_b \times n \times D_b}} = \sqrt[3]{\frac{8 \times 215000}{\pi \times 40 \times 4 \times 105}} = 5.708 \text{mm} \]

Check for crushing stress in the bolt material:

\[ T = \left[ (d_b \times t_f) \times \sigma_{cb} \times n \right] \times \frac{D_b}{2} \]

\[ \sigma_{cb} = \frac{2T}{(d_b \times t_f) \times n \times D_b} = \frac{2 \times 215000}{(5.708 \times 17.5) \times 4 \times 105} = 10.249 \text{ N/mm}^2; \]

Since \( \sigma_{cb} < 80 \text{ N/mm}^2 \)

(i.e. permissible crushing stress of the bolt material) hence designed nominal diameter of the bolt is safe and successful

Standardise the Bolt Refer the table no. 9.8 page no. 113,


\[ d_b = 6.0 \text{ or} \]

bolt M6.0 is to be used for the given application.

Q.1: Design a typical rigid flange coupling for connecting a motor and a centrifugal pump shafts. The coupling needs to transmit 15 KW at 1000 rpm. The allowable shear stresses of the shaft, key and bolt materials are 60 MPa, 50 MPa and 25 MPa respectively. The shear modulus of the shaft material may be taken as 84 GPa. The angle of twist of the shaft should be limited to 1 degree in 20 times the shaft diameter.

The shaft diameter based on strength may be given by

\[ d = \sqrt[3]{\frac{16T}{\pi \tau_y}} \text{ where } T \text{ is the torque transmitted and } \tau_y \text{ is the allowable yield stress in shear.} \]

Here \( T = \text{Power/} \left( \frac{2\pi N}{60} \right) = \frac{15 \times 10^3}{\frac{2\pi \times 1000}{60}} = 143 \text{ Nm} \)

And substituting \( \tau_y = 60 \times 10^6 \text{ Pa} \) we have

\[ d = \left( \frac{16 \times 143}{\pi \times 60 \times 10^6} \right)^\frac{1}{3} = 2.29 \times 10^{-2} \text{ m} = 23 \text{ mm}. \]
Let us consider a shaft of 25 mm which is a standard size.

From the rigidity point of view

\[ \frac{T}{J} = \frac{G\theta}{L} \]

Substituting \( T = 143 \text{Nm}, \ J = \frac{\pi}{32} (0.025)^4 = 38.3 \times 10^{-6} \text{m}^4, G = 84 \times 10^9 \text{Pa} \)

\[ \frac{\theta}{L} = \frac{143}{38.3 \times 10^{-6} \times 84 \times 10^9} = 0.044 \text{ radian per meter.} \]

The limiting twist is 1 degree in 20 times the shaft diameter

which is \( \frac{\pi}{180 \times 20 \times 0.025} = 0.035 \text{ radian per meter} \)

Therefore, the shaft diameter of 25mm is safe.

We now consider a typical rigid flange coupling as shown in Figure

**Hub**

Using empirical relations

Hub diameter \( d_1 = 1.75d + 6.5 \text{ mm} \). This gives

\[ d_1 = 1.75 \times 25 + 6.5 = 50.25 \text{ mm} \] say \( d_1 = 51 \text{ mm} \)

Hub length \( L = 1.5d \). This gives \( L = 1.5 \times 25 = 37.5 \text{ mm} \), say \( L = 38 \text{ mm} \)

Hub thickness \( t_1 = \frac{d_1 - d}{2} = \frac{51 - 25}{2} = 13 \text{ mm} \)

**Key** –

Now to avoid the shear failure of the key
Assuming an allowable crushing stress for the key material to be 100 MPa, the key design is safe. Therefore the key size may be taken as: a square key of 6.25 mm size and 37.5 mm long. However, keeping in mind that for a shaft of diameter between 22 mm and 30 mm a rectangular key of 8 mm width, 7 mm depth and length between 18 mm and 90 mm is recommended. We choose a standard key of 8 mm width, 7 mm depth and 38 mm length which is safe for the present purpose.

**Bolts.**

To avoid shear failure of bolts

\[
\frac{n \pi d_b^2 \sigma}{4} \frac{d_c}{2} = T
\]

where number of bolts \( n \) is given by the empirical relation...
\[ n = \frac{4}{150} d + 3 \] where \( d \) is the shaft diameter in mm.

which gives \( n = 3.66 \) and we may take \( n = 4 \) or more.

Here \( \tau_{yb} \) is the allowable shear stress of the bolt and this is assumed to be 60 MPa.

\( d_c \) is the bolt circle diameter and this may be assumed initially based on hub diameter \( d_1 = 51 \text{ mm} \) and later the dimension must be justified.

Let \( d_c = 65 \text{ mm} \).

Substituting the values we have the bolt diameter \( d_b \) as

\[ d_b = \left( \frac{8T}{n \pi \tau_{yb} \sigma_c} \right)^{\frac{1}{2}} \text{ i.e. } \left( \frac{8 \times 143}{4 \pi \times 25 \times 10^6 \times 65 \times 10^{-3}} \right)^{\frac{1}{2}} = 7.48 \times 10^{-3} \]

which gives \( d_b = 7.48 \text{ mm} \).

With higher factor of safety we may take \( d_b = 10 \text{ mm} \) which is a standard size.

We may now check for crushing failure as

\[ nd_b t_2 \sigma_c \frac{d_1}{2} = T \]

Substituting \( n = 4, \ d_b = 10 \text{ mm}, \sigma_c = 100 \text{ MPa}, \ d_c = 65 \text{ mm}, \ T = 143 \text{ Nm} \) and this gives \( t_2 = 2.2 \text{ mm} \).

However empirically we have \( t_2 = \frac{1}{2} t_1 + 6.5 = 13 \text{ mm} \)

Therefore we take \( t_2 = 13 \text{ mm} \) which gives higher factor of safety.

**Protecting flange thickness.**

Protecting flange thickness \( t_3 \) is usually less than \( \frac{1}{2} t_2 \) we therefore take \( t_3 = 8 \text{ mm} \) since there is no direct load on this part.
**Spigot depth**

Spigot depth which is mainly provided for location may be taken as 2mm.

**Check for the shear failure of the hub**

To avoid shear failure of hub we have

\[ \pi d t_s \frac{d}{2} = T \]

Substituting \( d_t = 51 \text{mm}, \ t_s = 13 \text{mm} \) and \( T = 143 \text{Nm} \), we have shear stress in flange \( \tau_f \) as

\[ \tau_f = \frac{2T}{(\pi d_t^2 t_s)} \]

And this gives \( \tau_f = 2.69 \text{ MPa} \) which is much less than the yield shear value of flange material 60 MPa.

Q. It is required to design square key for fixing a gear to transmit a torque of 198943.68 N-mm. The key is made of plain carbon steel having the yield pointy in tension and in compression as 460 MPa and factor of safety 3. Determine the dimensions of the key.

**Solution:**

Given \( S_{yt} = S_{yc} = 460 \text{MPa}, \ f.o.s = 3.0; \ \text{Torque} "T" = 198943.68 \text{N} \cdot \text{mm} \)

**Diameter of the shaft “d”:**

Applying the maximum shear stress theory

\[ \frac{16T}{\pi d^3} \leq \frac{S_{yt}}{2 \times f.o.s} \Rightarrow \frac{16 \times 198943.68}{\pi d^3} \leq \frac{460}{2 \times 3} \Rightarrow d \geq 23.64 \]

Taking the shaft diameter as

\( d = 25 \text{ mm} \).

**The proportion of the square key :**

\( w = h = \frac{d}{4} = \frac{25}{4} = 6.25 \text{mm} \approx 6.0 \text{mm} \)

Considering the shearing of the key the length of the key may be given as :

\[ l = \frac{2 \times T}{\tau \times d \times w} = \frac{2 \times 198943.68}{\left(\frac{460}{2 \times 3}\right) \times 25 \times 6} = 34.60 \text{ mm}; \]

Considering the crushing of the key the length of the key may be given as :

\[ l = \frac{4 \times T}{\sigma_c \times d \times w} = \frac{2 \times 198943.68}{\left(\frac{460}{3}\right) \times 25 \times 6} = 34.60 \text{ mm}; \]

Hence the length of the key is 35 mm;

The key is 6X6X35mm.
Q. It is required to design square key for fixing a gear to transmit a torque of 198943.68 N-mm. The key is made of plain carbon steel having the yield point in tension and in compression as 460 MPa and factor of safety 3. Determine the dimensions of the key.

Solution: Given \( S_{yt} = S_{yc} = 460\text{ MPa}, f.o.s = 3.0; \) Torque \( "T" = 198943.68\text{ N-mm} \)

**Diameter of the shaft “d”:**
Applying the maximum shear stress theory
\[
\frac{16T}{\pi d^3} \leq \frac{S_{yt}}{2 \times f.o.s} \Rightarrow \frac{16 \times 198943.68}{\pi d^3} \leq \frac{460}{2 \times 3} \Rightarrow d \geq 23.64
\]
Taking the shaft diameter as \( d = 25 \text{ mm} \).

**The proportion of the square key:**
\[
w = h = \frac{d}{4} = \frac{25}{4} = 6.25\text{ mm} \approx 6.0\text{ mm}
\]
Considering the shearing of the key the length of the key may be given as:
\[
l = \frac{2 \times T}{\tau \times d \times w} = \frac{2 \times 198943.68}{(\frac{460}{2 \times 3}) \times 25 \times 6} = 34.60\text{ mm};
\]
Considering the crushing of the key the length of the key may be given as:
\[
l = \frac{4 \times T}{\sigma_c \times d \times w} = \frac{2 \times 198943.68}{(\frac{460}{3}) \times 25 \times 6} = 34.60\text{ mm};
\]

Hence the length of the key is 35 mm;

The key is 6X6X35mm.

Q. A machine shaft is to transmit 61.5 kW at a speed of 1150 rev./min. with mild shocks. The shaft is subjected to a maximum bending moment of 900 N-m and an axial thrust of 70 kN. The shaft is supported at intervals of 2.5 meter. What should be its diameter when designed according to the ASME code and keyway effect is considered in the design?

Solution: According to code, shaft is made of ductile material whose ultimate tensile strength is twice the ultimate shear strength, and the shaft diameter is controlled by the maximum shear theory regardless of the ratio of the twisting moment to the bending moment.
\[
\tau = \left[ \frac{K_s 16 T}{\pi d^4} \right]^2 + \frac{1}{4} \left( \frac{K_b \times 32 M_b}{\pi d^4} + \alpha \frac{F_a}{\pi d^2} \right)^\frac{1}{2} ;
\]

Let
\( \tau = 56 \text{ MPa} ; \quad K_s = 1.5 ; \quad K_b = 1.5 ; \quad F_a = 70 \text{ kN} ; \quad M_b = 900 \text{ N.m} ; \quad & T = \frac{950 \times 61.5}{1150} = 510 \text{ N.m} ; \quad \)

Neglecting \( F_a \); the diameter of the shaft
\[
d^4 = \frac{16}{\pi \times \tau} \left[ (K_b M_b)^2 + (K_s T)^2 \right] = \frac{16}{\pi \times 56} \left[ (1.5 \times 900)^2 + (1.5 \times 510)^2 \right] ;
\]
\( d = 52 \text{ mm} ; \)

Let us try for \( d = 60 \text{ mm} ; \)
\[ \therefore \text{Radius of gyration, } k = \frac{d}{4} = \frac{60}{4} = 15 \text{ mm} ; \]
and \( \frac{L}{k} = \frac{2500}{15} = 166.7 > 115 \)
\[ \alpha = \frac{S_{yp} (L/k)^2}{c \pi^2 E} ; \]
Assume \( g \)
\[ S_{yp} = 300 \text{ MPa} ; \quad c = 1.6 \text{ for partially restrained ends;} \]
\[ \alpha = \frac{300 \times 166.7^2}{1.6 \pi^2 \times 210 \times 10^9} = 2.51 \]
Hence
\[
\tau = \left[ \frac{(1.5 \times 16 \times 510)}{\pi \times 0.60^3} \right]^2 + \frac{1}{4} \left( \frac{1.5 \times 32 \times 900}{\pi \times 0.60^3} + 2.51 \times \frac{70000}{\pi \times 0.60^3} \right)^\frac{1}{2} = 65.5 \text{ MPa} ;
\]

Since induced shear stress in the material of the shaft is more than the permissible shear stress i.e. 56 MPa hence the assumed dimension is unsafe.

Second trial
Let
\( d = 64 \text{ mm} ; \)
\[ \therefore \text{Radius of gyration, } k = \frac{d}{4} = \frac{64}{4} = 16 \text{ mm} ; \]
and \( \frac{L}{k} = \frac{2500}{16} = 156.25 > 115 \)
\[ \alpha = \frac{S_{yp} (L/k)^2}{c \pi^2 E} ; \]
Assume \( g \)
\[ S_{yp} = 300 \text{ MPa} ; \quad c = 1.6 \text{ for partially restrained ends;} \]
\[ \alpha = \frac{300 \times 156.25^2}{1.6 \pi^2 \times 210 \times 10^9} = 2.2086 \]
Hence
\[
\tau = \left[ \frac{(1.5 \times 16 \times 510)}{\pi \times 0.60^3} \right]^2 + \frac{1}{4} \left( \frac{1.5 \times 32 \times 900}{\pi \times 0.60^3} + 2.2086 \times \frac{70000}{\pi \times 0.60^3} \right)^\frac{1}{2} = 52.32 \text{ MPa} ;
\]
Since induced shear stress in the material of the shaft is less than the permissible shear stress i.e. 56 MPa with the diameter of 64 mm hence the assumed dimension is safe.

**DIAMETER OF THE SHAFT = 64 mm.**
Q. It is required to design a rigid type of flange coupling to connect two shafts. The input shaft transmits 37.4 kW power at 175 rpm to the output shaft through the coupling. The design torque is 1.5 times of the rated torque. Select suitable material for various parts of the coupling, design the coupling and specify the dimensions of its components.

Given: \( P = 37.4 \text{ kW} \); \( N = 175 \text{ r.p.m.} \); Service factor = 1.5

The torque is to be transmitted

\[
T = \frac{kW \times 60 \times 10^6}{2 \times \pi \times N} = \frac{37.4 \times 60 \times 10^6}{2 \times \pi \times 175} = 2040821.099 \text{ N-mm};
\]

Assume: Protective type rigid flange coupling

<table>
<thead>
<tr>
<th>Component</th>
<th>Material</th>
<th>Permissible stress</th>
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<tbody>
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<td>Shaft</td>
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<td>shear stress “( \tau )” = 40 MPa;</td>
</tr>
<tr>
<td>Key</td>
<td>Mild steel</td>
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<td>Bolt</td>
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</tr>
<tr>
<td>Flange</td>
<td>Cast iron</td>
<td>shear stress “( \tau )” = 8 MPa</td>
</tr>
</tbody>
</table>

1. Design for hub

Determining the diameter of the shaft \( d \) for the torque transmitted by the shaft, Since the service factor is 1.5, therefore the maximum torque transmitted by the shaft,

\[
T_{max} = 1.5 \times 2040821.099 = 3061231.65 \text{ N-mm};
\]

The torque transmitted by the shaft \( T \),

\[
T = \frac{\pi}{16} \times d^3 \times \tau_s = \frac{\pi}{16} \times d^3 \times 40 = 7.854d^3
\]

\[
.: 3061231.65 = 7.854d^3
\]

\[
d^3 = \frac{3061231.65}{7.854} = 389768.12
\]

\[
d = 73.05 \approx 75 \text{ mm}
\]

We know that outer diameter of the hub,

\[
D = 2d = 2 \times 75 = 150 \text{ mm} \quad \text{Ans.}
\]

and length of hub, \( L = 1.5 \times 75 = 112.5 \text{ mm} \quad \text{Ans.} \)
Let us now check the induced shear stress for the hub material which is cast iron. Considering the hub as a hollow shaft. We know that the maximum torque transmitted ($T_{max}$).

\[
3061231.65 = \frac{\pi}{16} \times \tau_c \left[ \frac{D^4 - d^4}{D} \right] = \frac{\pi}{16} \times \tau_c \left[ \frac{150^4 - 75^4}{150} \right]
\]

\[
\tau_c = \frac{3061231.65 \times 16 \times 150}{\pi \times (150^4 - 75^4)} = 4.93 \text{MPa};
\]

Since the induced shear stress for the hub material (i.e. cast iron) is less than the permissible value of 8 MPa, therefore the design of hub is safe.

**2. Design for key**

Assuming the square key is to be used and the crushing stress for the key material is twice its shear stress (i.e. $\sigma_{ck} = 2\tau_k$),

square key may be used. From Table 4.1, pp#60, Design Data Handbook for Mechanical Engineers by K. Mahadevan and K. Balveera Reddy, for a shaft of 75 mm diameter,

Width of key, $w = 22$ mm  **Ans.**

and thickness of key, $t = w = 22$ mm  **Ans.**

The length of key ($l$) is taken equal to the length of hub.

\[ l = L = 112.5 \text{ mm  Ans.} \]

Let us now check the induced stresses in the key by considering it in shearing and crushing. Considering the key in shearing. We know that the maximum torque transmitted ($T_{max}$),

\[
3061231.65 = l \times w \times \tau_k \times \frac{d}{2}
\]

\[
\tau_k = \frac{3061231.65 \times 2}{l \times w \times \tau_k \times d} = \frac{3061231.65 \times 2}{112.5 \times 22 \times 75} = 32.983 \text{ MPa};
\]

Considering the key in crushing. We know that the maximum torque transmitted ($T_{max}$),

\[
3061231.65 = \frac{l \times t \times \sigma_{ck} \times d}{2}
\]

\[
\sigma_{ck} = \frac{3061231.65 \times 4}{l \times t \times d} = \frac{3061231.65 \times 4}{112.5 \times 22 \times 75} = 65.97 \text{ MPa};
\]
Since the induced shear and crushing stresses in the key are less than the permissible stresses, therefore the design for key is safe.

3. Design for flange

The thickness of flange \((t_f)\) is taken as \(0.5 \, d\).

\[ t_f = 0.5 \, d = 0.5 \times 75 = 37.5 \text{ mm} \, \text{Ans.} \]

Check the induced shearing stress in the flange by considering the flange at the junction of the hub in shear. We know that the maximum torque transmitted \((T_{max})\),

\[ \tau_c = \frac{3061231 \times 0.65 \times 2}{\pi \times 150^2 \times 37.5} = 2.31 \text{ MPa; } \]

Since the induced shear stress in the flange is less than 8 MPa, therefore the design of flange is safe.

4. Design for bolts

Let \(d_1\) = Nominal diameter of bolts.

Since the diameter of the shaft is 75 mm, therefore let us take the number of bolts,

\[ n = 4 \quad \text{(Assumed)} \]

and pitch circle diameter of bolts,

\[ D_1 = 3d = 3 \times 75 = 225 \text{ mm} \]

The bolts are subjected to shear stress due to the torque transmitted. We know that the maximum torque transmitted \((T_{max})\),

\[ d_1^2 = \frac{3061231.65 \times 8}{\pi \times 40 \times 4 \times 225} = 216.54; \]
\[ d_1 = 14.71 \text{ mm} \]

Assuming coarse threads, the nearest standard size of bolt is M 18. \textbf{Ans.}

\textbf{(Ref. Table # 9.8, pp#114)}

Other proportions of the flange are taken as follows:
Outer diameter of the flange,

\[ D_2 = 4 \times d = 4 \times 75 = 300 \text{ mm} \ \textbf{Ans.} \]

Thickness of the protective circumferential flange,

\[ t_p = 0.25 \times d = 0.25 \times 75 = 18.75 \text{ mm} \ \text{say} \ 20 \text{ mm} \ \textbf{Ans.} \]
UNIT-IV

Mechanical Spring

Spring provide a flexible joint between two parts or bodies. Spring may be used to absorb the energy and control the motion of two mating bodies.

Basic Functions of Spring
1. Cushioning, absorbing, or controlling of energy due to shock and vibration. e.g. Car springs or railway buffers
2. To control energy, springs-supports and vibration dampers.
3. Control of motion, and maintaining contact between two elements (cam and its follower)
4. Creation of the necessary pressure in a friction device (a brake or a clutch)
5. Restoration of a machine part to its normal position when the applied force is withdrawn (a governor or valve). A typical example is a governor for turbine speed control. A governor system uses a spring controlled valve to regulate flow of fluid through the turbine, thereby controlling the turbine speed.
6. Measuring forces, spring balances, gages
7. Storing of energy, in clocks or starters, the clock has spiral type of spring which is wound to coil and then the stored energy helps gradual recoil of the spring when in operation.

Before considering the design aspects of springs we will have a quick look at the spring materials and manufacturing methods.

Materials Used for Spring:
One of the important considerations in spring design is the choice of the spring material. Some of the common spring materials are given below.

Hard-drawn wire:
This is cold drawn, cheapest spring steel. Normally used for low stress and static load. The material is not suitable at subzero temperatures or at temperatures above 1200C.

Oil-tempered wire:
It is a cold drawn, quenched, tempered, and general purpose spring steel. However, it is not suitable for fatigue or sudden loads, at subzero temperatures and at temperatures above 1800C. When we go for highly stressed conditions then alloy steels are useful.

Chrome Vanadium:
This alloy spring steel is used for high stress conditions and at high temperature up to 2200C. It is good for fatigue resistance and long endurance for shock and impact loads.
**Chrome Silicon:**
This material can be used for highly stressed springs. It offers excellent service for long life, shock loading and for temperature up to 250°C.

**Music Wire:**
This spring material is most widely used for small springs. It is the toughest and has highest tensile strength and can withstand repeated loading at high stresses. However, it can not be used at subzero temperatures or at temperatures above 120°C. Normally when we talk about springs we will find that the music wire is a common choice for springs.

**Stainless Steel:**
Widely used alloy spring materials.
Phosphor Bronze / Spring Brass: It has good corrosion resistance and electrical conductivity. That’s the reason it is commonly used for contacts in electrical switches. Spring brass can be used at subzero temperatures. Helical spring
The figures below show the schematic representation of a helical spring acted upon by a tensile load $F$ and compressive load $F$. The circles denote the cross section of the spring wire. The cut section, i.e. from the entire coil somewhere we make a cut, is indicated as a circle with shade.

![Figure 4.1](image-url)
SOME DEFINITIONS RELATED TO HELICAL SPRING:

**Solid Length**: The axial length of the spring under loaded condition when the coils of the spring touches each other is known as solid length of the spring as shown in below figure,

**Free Length**: The axial length of the spring under no loading condition is known as solid length of the spring as shown in below figure,

**Compressed Length**: The compressed length is defined as the axial length of the spring, that is subjected to maximum compressive force.

![Figure 4.2](image)

**Compressed Length** = **Free Length** - **Maximum deflection**

= \(N_t d + (N_t - 1) \times \text{gap between adjacent coils}\);

**Spring Index**: Spring index is defined as the ratio of the mean coil diameter to the wire diameter of the spring and denoted as “C”.

\[
C = \frac{D}{d} = \frac{\text{Mean coil diameter}}{\text{Wire diameter}}
\]

The spring index indicates the relative sharpness of curvature of the coil.

**Spring Rate**: Spring rate may be defined as the force per unit deflection of the spring, it is also known as stiffness constant.

**Types of Stresses Produced In The Wire of the Closed Coiled Helical Spring**:

In addition to the torsional shear stress \(\tau_1\) induced in the wire,

\[
\tau_1 = \frac{8W_D}{\pi d^3}
\]
The following stresses also act on the wire:

1. Direct shear stress due to the load $W$, and
2. Stress due to curvature of wire.

$$\tau_2 = \frac{\text{Load}}{\text{Cross-sectional area of the wire}} = \frac{W}{\frac{\pi}{4} \times d^2} = \frac{4W}{\pi d^2}$$

**The Resultant Shear Stress**

$$\tau = \tau_1 \pm \tau_2 = \frac{8WD}{\pi d^3} \pm \frac{4W}{\pi d^2}$$

The *positive* sign is used for the inner edge of the wire and *negative* sign is used for the outer edge of the wire. Since the stress is maximum at the inner edge of the wire, therefore

Maximum shear stress induced in the wire = Torsional shear stress + Direct shear stress

$$= \frac{8WD}{\pi d^3} + \frac{4W}{\pi d^2} = \frac{8WD}{\pi d^3} \left(1 + \frac{d}{2D}\right)$$

$$= \frac{8WD}{\pi d^3} \left(1 + \frac{1}{2C}\right) = K_s \times \frac{8WD}{\pi d^3}$$

$K_s = \text{Shear stress factor}$

In order to consider the effects of both direct shear as well as curvature of the wire, a Wahl’s stress factor ($K$) introduced by A.M. Wahl may be used.

$$\therefore \text{Maximum shear stress induced in the wire,}$$

$$\tau = K \times \frac{8WD}{\pi d^3} = K \times \frac{8WC}{\pi d^2}$$

where

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C}$$
Distribution of Stresses:

Figure 4.3

Here the “d” and “D” represents the wire diameter and mean coil diameter of the spring.

Q. A helical compression spring is made of oil tempered carbon steel and is subjected to a load varying from 600 N to 1000 N. The spring index is 6 and the factor of safety is 1.5. If the yield stress & endurance stress of the material is 700 MPa, and 350 MPa respectively. Find the wire diameter of the spring.

Solution:

Given:

\[ F_{\text{max}} = 1000 \text{N}; \quad F_{\text{min}} = 600 \text{N}; \quad \text{Spring index}'' C'' = 6.0; \quad \text{Factor of safety} :'' N_f'' = 1.5 \]
\[ S_{\text{sy}} = 700 \text{MPa}; \quad S'_{\text{se}} = 350 \text{MPa}; \]

1. **Mean Force and Amplitude Force**:

\[ F_m = \frac{F_{\text{max}} + F_{\text{min}}}{2} = \frac{1000 + 600}{2} = 800 \text{ N}; \]
\[ F_a = \frac{F_{\text{max}} - F_{\text{min}}}{2} = \frac{1000 - 600}{2} = 200 \text{ N}; \]
2. Mean Shear Stress:

Shear stress multiplication factor

\[ K_s = \left[ 1 + \frac{0.5}{C} \right] = \left[ 1 + \frac{0.5}{6.0} \right] = 1.0833; \]

\[ \tau_m = K_s \times \left( \frac{8F_mC}{\pi d^2} \right) = 1.0833 \times \left( \frac{8 \times 800 \times 6}{\pi d^2} \right) = \frac{13241.69}{d^2} \text{ N/mm}^2; \]

3. Amplitude Shear Stress:

Wahl's Correction Factor

\[ K_w = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 6 - 1}{4 \times 6 - 4} + \frac{0.615}{6} = 1.2525; \]

\[ \tau_a = K_w \times \left( \frac{8F_aC}{\pi d^2} \right) = 1.2525 \times \left( \frac{8 \times 200 \times 6}{\pi d^2} \right) = \frac{3827.35}{d^2} \text{ N/mm}^2; \]

4. Spring Wire Diameter:

\[ \frac{\tau_a}{S_{se}} = \frac{S_{sy}}{N_f} = \frac{\tau_m}{S_{sy}} - \frac{S'_{se}}{2} \Rightarrow \frac{\tau_a}{S_{sy}} = \frac{\frac{3827.35}{d^2}}{\frac{13241.69}{d^2}} = \frac{\frac{350}{2}}{700 - \frac{350}{2}} \]

\[ \frac{3827.35}{d^2} = \frac{175}{466.6667 - \frac{13241.69}{d^2}} \]

\[ \frac{3827.35}{d^2} = \frac{155.555}{4413.8967} \]

\[ \frac{8241.2467}{d^2} = 155.555 \]

\[ d^2 = 52.98; \]

\[ d = 7.28 \text{ mm or } 7.30 \text{ mm} \]

\[ d = 7.30 \text{ mm} \]

Q. A helical valve spring is to be designed for an operating load range of 90 N to 140 N. The 90 N load acts when the valve is closed and 140 N force acts when the valve is open. The deflection of the spring is limited to 8 mm. Take G=84 GPa.
Solution:

Given: \( W_1 = 90 \text{ N} \); \( W_2 = 140 \text{ N} \); \( \delta = 8 \text{ mm} \); \( G = 84 \text{ kN/mm}^2 = 84 \times 10^3 \text{ N/mm}^2 \);

Assume: Spring index \( C = \frac{D}{d} = 6 \)

Permissible shear stress intensity \( \tau = 420 \text{ MPa} = 420 \text{ N/mm}^2 \)

1. Mean Diameter of the Spring Coil

Let \( D \) = Mean diameter of the spring coil for a maximum load of \( W_2 = 140 \text{ N} \), and \( d \) = Diameter of the spring wire.

The twisting moment on the spring,

\[
T = \frac{W_2 \times D}{2} = \frac{140 \times 6d}{2} = 420d
\]

\[
420d = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 420 \times d^3 = 82.467d^3
\]

\[
d^2 = \frac{420}{82.467} = 5.093 \text{ mm};
\]

\[
d = 2.257 \text{ mm};
\]

Standard wire diameter \( d = 2.40 \text{ mm} \) (Refer: Table# 11.3a, pp#150)

\[\therefore\] Mean diameter of the spring coil,

\[
D = 6d = 6 \times 2.40 = 14.40 \text{ mm} \text{ Ans.}
\]

The outer diameter of the spring coil,

\[
D_o = D + d = 14.40 + 2.40 = 16.80 \text{ mm} \text{ Ans.}
\]

and inner diameter of the spring coil,

\[
D_i = D - d = 14.40 - 2.40 = 12.0 \text{ mm} \text{ Ans.}
\]

2. Number of Turns of the Spring Coil

Let \( n \) = Number of active turns.

It is given that the axial deflection (\( \delta \)) for the load range from 90 N to 140 N (i.e. for \( W = 50 \text{ N} \)) is 8 mm.

The deflection of the spring (\( \delta \)), may be given as
For squared and ground ends, the total number of turns,

\[ n' = 17 + 2 = 19 \text{ Ans.} \]

3. **Free Length of the Spring**

Since the compression produced under 50 N is 8 mm, therefore maximum compression produced under the maximum load of 140 N is

\[ \delta_{\text{max}} = \frac{8}{50} \times 140 = 22.4\text{mm}; \]

The free length of the spring,

\[ L_{F} = n'.d + \delta_{\text{max}} + 0.15 \delta_{\text{max}} = 19 \times 2.40 + 22.4 + 0.15 \times 22.4 \]

\[ = 71.36 \text{ say 72 mm Ans.} \]

4. **Pitch of the Coil**

The pitch of the coil

\[ p = \frac{L_{F}}{n' - 1} = \frac{72}{19 - 1} = 4\text{mm} \]
POWER SCREW:

A power screw is a drive used in machinery to convert a rotary motion into a linear motion for power transmission. It produces uniform motion and the design of the power screw may be such that

a. Either the screw or the nut is held at rest and the other member rotates as it moves axially. A typical example of this is a screw clamp.

b. Either the screw or the nut rotates but does not move axially. A typical example for this is a press.

Other applications of power screws are jack screws, lead screws of a lathe, screws for vices, presses etc.

Power screw normally uses square threads but ACME or Buttress threads may also be used. Power screws should be designed for smooth and noiseless transmission of power with an ability to carry heavy loads with high efficiency. We first consider the different thread forms and their proportions:

- Square Threads
- Acme Threads
- Trapezoidal Threads
- Buttress Threads

1. **Square Thread:** A square thread is shown in below figure and it is used transmission of power in both directions. It has maximum efficiency and minimum radial or bursting pressure on the nut. These threads are usually used in screw jack, mechanical presses and clamping devices

![Square Threads](image)

2. **Acme Threads:** In the following figure the acme threads are shown. Acme threads permits the use of split nut which can compensate the wear. And these threads are stronger than square threads, but having the efficiency lower than the square threads. These threads are usually used in lead screws.

![Acme Threads](image)
3. **Trapezoidal Threads:** In the following figure the trapezoidal threads are shown. These threads are similar to the acme threads except the thread angle which is $30^0$ in trapezoidal threads.

4. **Buttress Threads:** In the following figure the buttress threads are shown. It is used for transmission of power in one direction only. These threads are stronger in shear than any other power threads because of the largest cross section at the root. Buttress threads combine the high efficiency of square threads and high strength of V threads. These threads are used in vices and screw jacks where force is to be applied only in one direction.

**Efficiency Of A Power Screw:**
A square thread power screw with a single start is shown in the following figure. Here $p$ is the pitch, $\alpha$ helix angle, $d_m$ the mean diameter of thread and $F$ is the axial load. A developed single thread is shown in figure where $L = n \cdot p$ for a multi-start drive, $n$ being the number of starts. In order to analyze the mechanics of the power screw we need to consider two cases:
(a) Raising the load
(b) Lowering the load.
Raising the Load:

This requires an axial force $P$ as shown in the following figure. Here $N$ is the normal reaction and $\mu N$ is the frictional force.

For equilibrium

$$P - \mu N \cos \alpha - N \sin \alpha = 0$$

$$F + \mu N \sin \alpha - N \cos \alpha = 0$$

This gives

$$N = \frac{F}{(\cos \alpha - \mu \sin \alpha)}$$

$$P = \frac{F(\mu \cos \alpha + \sin \alpha)}{(\cos \alpha - \mu \sin \alpha)}$$
Torque transmitted during raising the load is then given by

\[ T_R = \frac{P d_m}{2} = \frac{F d_m}{2} \left( \frac{\mu \cos \alpha + \sin \alpha}{\cos \alpha - \mu \sin \alpha} \right) \]

Since \( \tan \alpha = \frac{L}{\pi d_m} \) we have

\[ T_R = \frac{F d_m}{2} \left( \frac{\mu \pi d_m + L}{\pi d_m - \mu L} \right) \]

The force system at the thread during lowering the load is shown in following figure. For equilibrium

\[ P - \mu N \cos \alpha + N \sin \alpha = 0 \]
\[ F - N \cos \alpha - \mu N \sin \alpha = 0 \]

This gives

\[ N = F \left( \frac{1}{\cos \alpha + \mu \sin \alpha} \right) \]
\[ p = \frac{F (\mu \cos \alpha - \sin \alpha)}{(\cos \alpha + \mu \sin \alpha)} \]

Torque required to lower the load is given by

\[ T_L = \frac{P d_m}{2} = \frac{F d_m}{2} \left( \frac{\mu \cos \alpha - \sin \alpha}{\cos \alpha + \mu \sin \alpha} \right) \]

And again taking \( \tan \alpha = \frac{L}{\pi d_m} \) we have

\[ T_L = \frac{F d_m}{2} \left( \frac{\mu \pi d_m - L}{\pi d_m + \mu L} \right) \]

Self Locking and Overhauling of Power Screw;

The torque required to lower the load is given as follows

\[ T_t = \frac{W d_m}{2} \left[ \tan(\phi - \lambda) \right] \]
Self Locking:

In above equation if $\phi > \lambda$, the torque required to lower the load $T_1$ will be positive. Such a screw is known as self locking screw.

\[ \phi > \lambda \]

\[ \tan \phi > \tan \lambda \Rightarrow \mu > \tan \lambda \] or \[ \mu > \frac{\text{lead}}{\pi \times d_m} \]

Over Hauling:

In above equation if $\phi < \lambda$, the torque required to lower the load $T_1$ will be negative. That is load will start to moving downward without the application of any torque. Such a screw is known as over hauling screw.

Efficiency of the power screw is given by

\[ \eta = \frac{\text{Work output}}{\text{Work input}} \]

Here work output = F. L

Work input = p. $\pi d_m$

This gives

\[ \eta = \frac{F}{p} \tan \alpha \]

Q. An electric motor driven screw moves a nut in a horizontal plane against a force of 75 kN at a speed of 250 mm/min. The screw has a single square thread of 6 mm pitch on a major diameter of 40 mm. The coefficient of friction at screw thread is 0.1. Estimate Power of the motor.

Solution:

The mean diameter of the screw $d_m$ = $d - 0.5p = 40 - 0.5 \times 6 = 37 mm; $

The lead angle $\lambda$ = $\tan^{-1} \left( \frac{L}{\pi \times d_m} \right);$ Where $L$ = lead of the screw.

$\therefore \lambda = \tan^{-1} \left( \frac{6}{\pi \times 37} \right) = 2.95^0$

Angle of friction $\phi = \tan^{-1} \mu = \tan^{-1} 0.1 = 5.71^0; $

The torque required to overcome the thread friction is given as

\[ T = \frac{W \times d_m}{2} \times \tan(\lambda + \phi) = \frac{75000 \times 37}{2} \times \tan(2.95^0 + 5.71^0) = 21132.1017 N - m \]
Hence the effort required

\[ F_1 = \frac{2 \times T}{d_m} = \frac{2 \times 211326.1017}{d_m} = 11423.03252 \text{N}; \]

The tangential speed

\[ v = \frac{250 \text{mm} \, \text{min}}{60} = \frac{0.250}{60} \text{m/s} = 0.0042 \text{m/s}; \]

Power of the motor required

\[ P = F_1 \times v = 11423.03252 \times 0.0042 = 47.595 \text{Watt} \approx 50 \text{Watt}. \]
SCREW JACK

Screw jack is a portable device consisting of a screw mechanism used to raise or lower the load. There are two types of jacks most commonly used,

1. Hydraulic
2. Mechanical

Merits of Screw jack –

1. Can be used to lift a heavy load against gravity.
2. Load can be kept in lifted position.
3. Due to leverage obtained by handle force required to raise load is very less & can be applied manually also

Demerits of screw jack –

1. Chances of dropping of load
2. Tipping or slipping of load.
3. This failure is not “SAFE FAIL” & can cause serious accidents.

Reasons of Accidents –

1. Load is improperly secured on jack.
2. The screw jack is overloaded.
3. Center of gravity is off center with axis of jack.
4. Jack is not placed on hard & level surface.
5. Using for other purpose instead of using it for which it is designed.

Screw jack has following parts :-

1. Frame
2. Screw
3. Nut
4. Handle
5. Cup
6. Set Screw
7. Washer

(1) Frame
Frame Size - Most of the times frame is conical in shape and hollow internally to accommodate a nut & screw assembly. The hollow conical shape insures a safe & complete resting of a jack on ground. If it is provided with legs like structure, it quite possible that in case of uneven distribution load may fail down because all legs will not touch ground.
**Force Analysis** – The force by a load is directed by a cup to screw then is directed by cup to screw then to threads of screw to nut then to frame so it is always compressive in nature.

**Manufacturing Process** – The complex shape of frame leads us to use a ‘Casting’ process for manufacturing. For all this purpose We need to select a cast iron as material for frame. We select a FG200 as material for frame such as it contains carbon precipitates as “graphite flakes” as graphite is soft in nature it improves its ability to resist a compressive load.

(2) **Screw** –
- **Screw size** – Screws is nothing but a member having Helical groove around periphery of solid bar. It can be around 22 to 100mm diameter for square power screws & 24 to 100mm for trapezoidal power screws.
- **Thread profile** – The screw or power screw thread is always a square type because it has more efficiency than trapezoidal threads and there is no radial thrust on screw i.e. no Bursting Pressure, so motion is uniform.
- **Square threads** usually turned on lathes using single point cutting tool. It leads us to use free cutting steel.
- **Square threads are weak in roots. Wear of thread surface lead us to use “Unalloyed free cutting steel”**.

(3) **Nut** –
As we know there always a relative motion between screw and nut, which cause a friction. The friction causes wear if some material is used for screw & nut it will wears both components. So one out of two has to be softer than other so as to ease of replacement. The size & shape of screw is costlier than nut, so generally we use softer material for nut than screw. Phosphor bronze is ideal material for nut which is a copper alloy having 0.2%phospher which increases tensile strength. Ultimate tensile strength for this is 190mpa and coefficient of friction is 0.1 Bearing pressure is 10mpa.
Advantages of phosphor bronze are,
(1) Good corrosion resistance.
(2) Low coefficient of friction
(3) Higher tensile strength than copper brass.

4] **Handle** –
Handle is subjected ti bending moments. So plain carbon steel with 0.3%carbon i.e. 30C8 can be selected. Yield strength in tension is 400mpa

(5) **Cup** –
Shape of cup is again complex and so economical to manufacture by Casting process, hence material will be cast iron with grade FG200.
(6) Set Screw –
Purpose of set screw is to resist motion of nut with screw. It can be of commercial steel.

(7) Washer –
Washer is to provide uniform force of tightening nut over screw force by enlarging area under actions of force. We can use commercial steel.
Procedure for the design of the screw having square threads: Threaded screw is subjected to compressive loads when used in the power screws.

1. The core diameter of the threaded screw spindle determined by assuming the pure direct compression as given below

\[
d_c = \sqrt[4]{\frac{4W}{\pi \times \sigma_c}};
\]

Where
- \(d_c\) = core diameter of the screwed spindle in mm;
- \(W\) = load applied in N;
- \(\sigma_c\) = permissible compressive stress of the threaded screw material in N/mm\(^2\);

2. Calculating the torque required to overcome the thread friction and the total torque required to raise the load as given
\[ T_t = \frac{Wd_m}{2} \left[ \tan(\phi + \lambda) \right] \]

and

\[ T_c = \mu_c \times W \times R_f \]

Where

\( \mu_c \) = coefficient of friction between the head and cup;

\( d_m \) = mean diameter of the screw = root diameter + \( \frac{\text{pitch of thread}}{2} \)

\( \phi \) = angle of friction between the threads of nut and screw

\( \lambda \) = lead angle of the screw thread

\( R_f \) = radius of friction circle

Total torque

\[ T = T_c + T_t \]

3. Then the diameter of the screw is checked against the induced torsional shear stress and principal stress induced due to the combined effect of torsion and bending moment

\[ \tau = \frac{16T_t}{\pi \times d_c^2} \]

screw is subjected to the bending moment \( M \)

induced bending stress

\[ \sigma_b = \frac{32 \times M}{\pi \times d_c^2} \]

Net Direct stress \( \sigma_d \) = bending stress + direct compressive stress

Maximum shear stress :

\[ \tau_{max} = \sqrt{\left( \frac{\sigma_d}{2} \right)^2 + \tau^2} \]

Maximum principal stress

\[ \sigma_1 = \frac{\sigma_d}{2} + \sqrt{\left( \frac{\sigma_d}{2} \right)^2 + \tau^2} \]

For successful design of the screw the maximum shear stress and maximum principal stresses must be less than the permissible respective stresses.

and finally the design will be checked against buckling using J.B. Johnson formula

\[ Crippling load \ W_c = \frac{\pi \times d_c^2}{4} \times S_{yc} \left[ 1 - \frac{S_{yc} (L/k)^2}{4C \pi^2 E} \right] \]

Symbols are having the usual meaning.

For successful design of the screw the crippling load must be greater than the applied load.
Design procedure of Nut: The following procedure is followed in designing of the nut of a screw jack;

1. **Height of the Nut:**
   The height of the nut is decided as follows:
   By considering the bearing failure of the nut threads the number of nut threads in mesh is calculated. Once the number of nut threads in mesh is known then the height of the nut will be determined as
   \[ \text{Height of the nut} = \text{number of nut threads in mesh} \times \text{pitch of thread}; \]
   In the next step these number of threads will be cheked against the direct shear stresses in the nut threads and screw theads. If fails then number of threads will be further increased to make it successful.

2. **The body diameter of the nut is determined by considering the tensile failure of the material of the nut.**
3. **The outside diameter of the collar of the nut is determined by considering the crushing failure of the material of the nut.**
4. **The thickness of the collar may be determined by considering the direct shear failure of the material of the nut.**

**Q.** Name the various components of the screw jack and their usual materials. A single start square threaded screw of mean diameter 24 mm and pitch of 5 is tightening by screwing a nut whose mean diameter at bearing surface is 50 mm. If the coefficient of friction between the nut and screw is 0.1 and for the nut and bearing surface is 0.16. Find the force required at the end of a spanner 0.5 meter long when the load on the screw is 10 kN.

**Ans.:**

<table>
<thead>
<tr>
<th>Name of the components of screw Jack</th>
<th>Material</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frame</td>
<td>FG200;</td>
</tr>
<tr>
<td>Screw</td>
<td>Plain Carbon Steel or Alloy Steel;</td>
</tr>
<tr>
<td>Nut</td>
<td>Phosphor Bronze;</td>
</tr>
<tr>
<td>Handle</td>
<td>Plain Carbon Steel or Alloy Steel;</td>
</tr>
</tbody>
</table>

**GIVEN:**
- Single start threaded screw: mean diameter “d_m” = 24 mm; Pitch “P” = 5 mm;
- Mean Diameter of the bearing surface “D_f” = 50 mm or Mean Radius of the bearing surface “R_f” = 25 mm; Coefficient of friction between the nut and screw “\( \mu_n \)” = 0.1;
- Coefficient of friction between the nut and bearing surface “\( \mu_c \)” = 0.16; Load “W” = 10 kN; Length of the spanner L = 0.5 meter = 500 mm.
Sol.:

\[
\lambda = \tan^{-1}\left(\frac{\text{Lead}}{\pi \times d_n}\right) = \tan^{-1}\left(\frac{5}{\pi \times 24}\right) = 3.794^\circ;
\]

Because for single start thread Lead = Pitch

Angle of friction between nut and thread \( \phi = \tan^{-1}(\mu_n) = \tan^{-1}(0.1) = 5.7106^\circ; \)

Torque required to overcome the thread friction between the nut and screw to raise the load "W"

\[
T_i = W \times \frac{d_n}{2} \times \tan(\lambda + \phi) = 10000 \times \frac{24}{2} \times \tan(3.794 + 5.7106) = 20091.00246 \text{ N-mm};
\]

Torque required to overcome the bearing surface friction between the nut and bearing surface raise the load "W"

\[
T_c = \mu_c \times W \times R_f = 0.16 \times 10000 \times 25 = 40000 \text{ N-mm};
\]

Total torque required to raise the given load

\[
T = T_i + T_c = 20091.00246 + 40000 = 60091.00246 \text{ N-mm};
\]

The force "P" required at the end of the spanner to raise the given load

\[
T = P \times L \Rightarrow 60091.00246 = P \times 500
\]

Hence \( P = \frac{60091.00246}{500} = 120.182 \text{ N} \approx 121 \text{ N} \) ANS.

Q. Design a screw jack for lifting a load of 20 kN through 200 mm.

Solution: Given: \( W = 20 \text{ kN} = 20000 \text{N} ; H_1 = 200 \text{ mm} = 0.2 \text{ m} ; \)

Assuming: Elastic strength of screw material in tension and compression

\[ \sigma_{et} = \sigma_{ec} = 200 \text{ MPa} \text{ and in shear } \tau_{et} = 120 \text{ MPa}; \]

The material for the nut is phosphor bronze; for which the elastic limit in

Tension \( \sigma_{et(nut)} = 100 \text{ MPa}; \)

Compression \( \sigma_{ec} (nut) = 90 \text{ MPa}; \)

Shear \( \tau_{e(nut)} = 80 \text{ MPa}; \)

Bearing pressure for nut \( p_b = 18 \text{N/mm}^2 ; \)

1. Design of screw for spindle

Let \( d_c = \text{Core diameter of the screw.} \)

Since the screw is under compression, therefore load \( (W), \)
For square threads of normal series, the following dimensions of the screw are selected from Table # 9.10, pp.#121.

Core diameter, \( d_c = 17 \) mm \textbf{Ans.} \\
Nominal or outside diameter of spindle, \( d_o = 22 \) mm \textbf{Ans.} \\
Pitch of threads, \( p = 5 \) mm \textbf{Ans.}

Now let us check for principal stresses:

We know that the mean diameter of screw,
\[
d = \frac{d_o + d_c}{2} = \frac{22 + 17}{2} = 19.5 \text{mm}
\]

\textbf{The helix angle}

\[
\tan \alpha = \frac{p}{\pi \times d} = \frac{5}{\pi \times 19.5} \\
\alpha = 4.67^0
\]

Assuming coefficient of friction between screw and nut is 0.14,
\[
\mu = \tan \phi = 0.14
\]

\[
\therefore \text{Torque required to rotate the screw in the nut,}
T_1 = P \times \frac{d}{2} = W \tan(\alpha + \phi) \frac{d}{2} = W \left[ \frac{\tan \alpha + \tan \phi}{1 - \tan \alpha \tan \phi} \right] \frac{d}{2} \\
= 20000 \times \left[ \frac{0.0817 + 0.14}{1 - 0.0817 \times 0.14} \right] \times 19.5^2 = 43729.31 \text{N} \cdot \text{mm};
\]

Now the compressive stress due to axial load
\[
\sigma_c = \frac{4 \times W}{\pi \times d_c^2} = \frac{4 \times 20000}{\pi \times 17^2} = 88.1135 \text{MPa};
\]

And shear stress due to torque
\[
20 \times 10^3 = \frac{\pi \times d_c^2 \times \sigma_{ce}}{f.o.s.} = \frac{\pi \times d_c^2 \times 200}{2.0}
\]
\[
d_c^2 = \frac{20000 \times 8}{\pi \times 200} = 254.65 \\
d_c = 15.95 \text{mm};
\]
\text{(Taking Factor of Safety = 2.0)}
\[ \tau = \frac{16 \times T_1}{\pi \times d_c^2} = \frac{16 \times 43729.31}{\pi \times 17^3} = 45.3311 \text{MPa}; \]

\( \therefore \) Maximum principal stress (tensile or compressive),

\[ \sigma_{\text{max}} = \frac{1}{2} \left[ \sigma_c + \sqrt{\sigma_c^2 + 4\tau^2} \right] = \frac{1}{2} \left[ 88.1135 + \sqrt{88.1135^2 + 4 \times 45.3311^2} \right] = 107.27 \text{MPa} \]

The given value of \( \sigma_c \) is equal to 200 /2=100 MPa, Hence the design is fail. Taking \( d_c=19 \text{mm} \); from Table # 9.10, pp.#121.

Core diameter, \( d_c = 19 \text{ mm} \) \textbf{Ans.}  

Nominal or outside diameter of spindle, \( d_o = 24 \text{ mm} \) \textbf{Ans.}  

Pitch of threads, \( p = 5 \text{ mm} \) \textbf{Ans.} 

Now let us check for principal stresses:

We know that the mean diameter of screw,

\[ d = \frac{d_o + d_c}{2} = \frac{24 + 19}{2} = 21.5 \text{ mm} \]

\textbf{The helix angle} 

\[ \tan \alpha = \frac{p}{\pi \times d} = \frac{5}{\pi \times 21.5} \]  
\[ \alpha = 4.23^\circ \]

Assuming coefficient of friction between screw and nut is 0.14,

\[ \mu = \tan \phi = 0.14 \]

\( \therefore \) Torque required to rotate the screw in the nut,

\[ T_1 = \frac{P \times d}{2} = \frac{W \tan(\alpha + \phi) \times d}{2} = \frac{W \left[ \frac{\tan \alpha + \tan \phi}{1 - \tan \alpha \tan \phi} \right] d}{2} \]
\[ = 20000 \times \left[ \frac{0.0704 + 0.14}{1 - 0.0704 \times 0.14} \right] \times \frac{21.5}{2} = 46474.95 \text{ N} \cdot \text{mm}; \]

Now the compressive stress due to axial load

\[ \sigma_c = \frac{4 \times W}{\pi \times d_c^2} = \frac{4 \times 20000}{\pi \times 19^2} = 70.54 \text{ MPa}; \]

And shear stress due to torque
\[
\tau = \frac{16 \times T_1}{\pi \times d_c^2} = \frac{16 \times 46474.95}{\pi \times 19^3} = 34.51 \text{ MPa};
\]

\[
\therefore \text{ Maximum principal stress (tensile or compressive)}, \quad \sigma_{c_{\text{max}}} = \frac{1}{2} \left[ \sigma_c + \sqrt{\sigma_c^2 + 4\tau_c^2} \right] = \frac{1}{2} \left[ 70.54 + \sqrt{70.54^2 + 4 \times 34.51^2} \right] = 84.615 \text{ MPa}
\]

And Maximum shear stress

\[
\tau_{c_{\text{max}}} = \frac{1}{2} \left[ \sqrt{\sigma_c^2 + 4\tau_c^2} \right] = \frac{1}{2} \left[ \sqrt{70.54^2 + 4 \times 34.51^2} \right] = 49.345 \text{ MPa}
\]

The given value of \(\sigma_c\) is equal to 200 \(\div 2\) = 100 MPa and \(\tau_c\) is equal to 120 \(\div 2\) = 60 MPa

Since these maximum stresses are within limits, therefore design of screw for spindle is safe. And the designed dimensions for the spindle are

Core diameter, \(d_c = 19 \text{ mm}\) \textbf{Ans.}

Nominal or outside diameter of spindle, \(d_o = 24 \text{ mm}\) \textbf{Ans.}

Pitch of threads, \(p = 5 \text{ mm}\) \textbf{Ans.}

\section*{2. Design for nut}

Let \(n = \) Number of threads in contact with the screwed spindle,

\[h = \text{Height of nut} = n \times p,\]  

and

\[t = \text{Thickness of screw} = \frac{p}{2} = \frac{5}{2} = 2.5 \text{ mm}\]

Assume that the load is distributed uniformly over the cross-sectional area of nut.

We know that the bearing pressure (\(p_b\)),

\[
18 = \frac{4 \times W}{\pi \times \left[ d_o^2 - d_c^2 \right] \times n} = \frac{4 \times 20000}{\pi \times \left[ 24^2 - 19^2 \right] \times n}
\]

\[
n = \frac{4 \times 20000}{\pi \times \left[ 24^2 - 19^2 \right] \times 18} = 6.58 \approx 7 \text{ threads}
\]

Now the height of THE NUT

\[h = n \times p = 7 \times 5 = 35 \text{ mm}\]

Now check the stresses induced in the screw and nut
\[ \tau_{\text{screw}} = \frac{W}{\pi \times n \times d_c \times t} = \frac{20000}{\pi \times 7 \times 19 \times 2.5} = 19.15 \text{ MPa} \]

And the stress in the nut

\[ \tau_{\text{nut}} = \frac{W}{\pi \times n \times d_o \times t} = \frac{20000}{\pi \times 7 \times 24 \times 2.5} = 15.16 \text{ MPa} \]

The permissible shear stress for the nut 80/2=40 MPa;

Since these stresses are within permissible limit, therefore design for nut is safe.

Let \( D_1 \) = Outer diameter of nut,

\( D_2 \) = Outside diameter for nut collar, and

\( t_1 \) = Thickness of nut collar.

First of all considering the tearing strength of nut, we have

\[ W = \frac{\pi}{4} \left[ D_1^2 - d_o^2 \right] t_1 \]

\[ 20000 = \frac{\pi}{4} \left[ D_1^2 - 24^2 \right] \left[ \frac{100}{2} \right] = 39.27 \times \left[ D_1^2 - 576 \right] \]

\[ D_1^2 - 576 = \frac{20000}{39.27} = 509.30 \]

\[ D_1 = 32.94 \approx 33 \text{ mm} \]

Now considering the crushing of the collar of the nut, we have

\[ W = \frac{\pi}{4} \left[ D_2^2 - D_1^2 \right] t_2 \]

\[ 20000 = \frac{\pi}{4} \left[ D_2^2 - 33^2 \right] \left[ \frac{90}{2} \right] = 35.343 \times \left[ D_2^2 - 1089 \right] \]

\[ D_2^2 - 1089 = \frac{20000}{35.343} = 565.883 \]

\[ D_2 = 40.68 \approx 41 \text{ mm} \]

Considering the shearing of the collar of the nut, we have

\[ W = \pi \times D_1 \times t_1 \times \tau \]

\[ 20000 = \pi \times 33 \times t_1 \times 40 \]

\[ t_1 = \frac{20000}{\pi \times 33 \times 40} = 4.823 \text{ mm} \approx 5 \text{ mm} \]

3. Design for handle and cup
The diameter of the head \((D3)\) on the top of the screwed rod is usually taken as 1.75 times the outside diameter of the screw \((do)\).

\[
\therefore \quad D3 = 1.75 \times do = 1.75 \times 24 = 42 \text{ say } 45 \text{ mm} \quad \textbf{Ans.}
\]

The head is provided with two holes at the right angles to receive the handle for rotating the screw. The seat for the cup is made equal to the diameter of head, \textit{i.e.} 45 mm and it is given chamfer at the top. The cup prevents the load from rotating. The cup is fitted to the head with a pin of diameter \(D4 = D3/4 = 45/4 = 11.25\) or 12 mm. The pin remains loose fit in the cup. Now let us find out the torque required \((T2)\) to overcome friction at the top of the screw.

Assuming uniform pressure conditions, we have

\[
T2 = \frac{2}{3} \mu_1 W \left[ \frac{R_3^2 - R_4^2}{R_3^2 - R_4^2} \right] = \frac{2}{3} \times 0.14 \times 20000 \left[ \frac{(45/2)^3 - (12/2)^3}{(45/2)^2 - (12/2)^2} \right] = 44357.895 \text{ N-mm} \quad \text{(Assuming } \mu_1 = \mu = 0.14)\]

Hence the total torque to which the handle is subjected,

\[
T = T1 + T2 = 46474.95 + 44357.895 = 90832.845 \text{ N-mm}
\]

Assuming that a force of 300 N is applied by a person intermittently, therefore length of handle required. Assuming the effort applied by an average person is 300 N

\[
= 90832.845 / 300 = 302.7762 \text{ mm say 305 mm.}
\]

Allowing some length for gripping, we shall take the length of handle as 305 mm. A little consideration will show that an excessive force applied at the end of lever will cause bending. Considering bending effect, the maximum bending moment on the handle,

\[
M = \text{Force applied } \times \text{Length of lever} = 300 \times 305 = 91500 \text{ N-mm}
\]

Let \(D = \text{Diameter of the handle.}\)

Assuming that the material of the handle is same as that of screw, therefore taking bending stress \(\sigma_b = \sigma_t = \sigma_{et} / 2 = 100 \text{ N/mm}^2\). The bending moment \((M)\),

\[
91500 = \frac{\pi}{32} \times \sigma_b \times D^3 = \frac{\pi}{32} \times 100 \times D^3 = 9.82D^3
\]

\[
D^3 = \frac{91500}{9.82} = 9317.72
\]

\[
D = 21.043 \approx 22 \text{ mm};
\]

The height of head \((H)\) is taken as \(2D\).
\[ H = 2 \quad D = 2 \times 22 = 44 \text{ mm} \textbf{Ans.} \]

Now let us check the screw for buckling load. The effective length for the buckling of screw,

\[ L = \text{Lift of screw} + 0.5 \times \text{height of nut} = \frac{H_1 + h}{2} \]

\[ = 200 + 44 / 2 = 221 \text{ mm} \]

When the screw reaches the maximum lift, it can be regarded as a strut whose lower end is fixed and the load end is free. We know that critical load,

\[ W_{cr} = A_c \times \sigma_y \times \left[ 1 - \frac{\sigma_y}{4\pi^2 E} \left( \frac{L}{k} \right)^2 \right] \]

For one end fixed and other end free \( C = 0.25 \);

Also \( k = 0.25d_c = 0.25 \times 19 = 4.75 \)

\[ \therefore \quad W_{cr} = \frac{\pi}{4} \times 19^2 \times 200 \times \left[ 1 - \frac{200}{4 \times 0.25 \times \pi^2 \times 210 \times 10^3 \times \left( \frac{221}{4.75} \right)^2} \right] \]

\[ W_{cr} = \frac{\pi}{4} \times 19^2 \times 200 \times [1 - 0.209] = 44854.25 \text{N}; \]

Since the critical load is more than the load at which the screw is designed (\( i.e. \ 20 \times 10^3 \) N), therefore there is no chance of the screw to buckle.

\textbf{4. Design of body}

The various dimensions of the body may be fixed as follows:

Diameter of the body at the top,

\[ D5 = 1.5 \quad D2 = 1.5 \times 41 = 61.5 \text{ mm} \textbf{Ans.} \]

Thickness of the body,

\[ t3 = 0.25 \ \text{do} = 0.25 \times 24 = 6 \text{ mm} \textbf{Ans.} \]

Inside diameter at the bottom,

\[ D6 = 2.25 \quad D2 = 2.25 \times 41 = 92.25 \text{ mm} \textbf{Ans.} \]

Outer diameter at the bottom,

\[ D7 = 1.75 \quad D6 = 1.75 \times 92.25 = 161.44 \text{ say } 165 \text{ mm} \textbf{Ans.} \]

Thickness of base, \( t2 = 2 \ t1 = 2 \times 5 = 10 \text{ mm} \textbf{Ans.} \)

Height of the body = Max. lift + Height of nut + 100 mm extra

\[ = 200 + 35 + 100 = 335 \text{ mm} \textbf{Ans.} \]
The body is made tapered in order to achieve stability of jack.

Let us now find out the efficiency of the screw jack. We know that the torque required to rotate the screw with no friction,

\[ T_o = W \tan \alpha \times \frac{d}{2} = 20000 \times \tan 4.234 \times \frac{21.5}{2} = 15916.8903 \text{ N-mm}; \]

Hence efficiency of the screw jack,

\[ \eta = \frac{T_o}{T} = \frac{15916.8903}{90832.845} = 0.175 = 17.5\% \]

Hence the designed screw jack offers self locking that is desirable.
REFERENCES